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Measurement of the growth rate of structures with type Ia
supernovae from the ZTF photometric survey

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Affidavit

I, undersigned, Bastien Carreres, hereby declare that the work presented in this manuscript is my own work, carried out under the scientific supervision of Dominique Fouchez and Benjamin Racine, in accordance with the principles of honesty, integrity and responsibility inherent to the research mission. The research work and the writing of this manuscript have been carried out in compliance with both the french national charter for Research Integrity and the Aix-Marseille University charter on the fight against plagiarism.

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List of publications and conference participation

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3. Euclid summer school 2020, 2021 and 2022

Résumé (French summary)

The acceleration of the expansion of the Universe is currently modeled by the presence of a dark energy represented by a cosmological constant Λ , which does not have an entirely satisfactory physical interpretation. The main alternatives to this constant are dynamical dark energy models or modifications to the laws of general relativity. Measuring the growth rate of structures $f\sigma_8$ through the observation of the velocity field of galaxies is an efficient way to detect possible deviations from the standard model and to discriminate between dynamical dark energy and modified gravity. In this thesis, I study the use of distance indicators such as type Ia supernovae to measure $f\sigma_8$. My work focuses on the development of a simulation of the ZTF survey to study the biases and systematics of the $f\sigma_8$ analysis. This analysis relies on the use of a velocity estimator that uses the measurements of a distance modulus of a SN Ia and of the *redshift* of the associated host galaxy. The velocities are then used in the maximization of a likelihood function of which $f\sigma_8$ is a free parameter. A simulation of 27 realizations of ZTF equivalent to six years of survey shows that the magnitude limit for the spectroscopic classification of SNe Ia leads to a selection bias after a *redshift* $z \sim 0.06$. This bias propagates onto $f\sigma_8$. Limiting the data with a cutoff at $z \sim 0.06$ allows an unbiased measurement of $f\sigma_8$ with a precision of 19%. This result is already comparable to results obtained with low-*redshift* galaxies. Furthermore, a perfect bias correction to include SNe Ia at higher *redshift* does not significantly reduce the error on $f\sigma_8$.

Abstract

The acceleration of the expansion of the Universe is modeled by the presence of dark energy, represented by a cosmological constant Λ , for which there is no entirely satisfactory physical interpretation. The main alternatives to this constant are models of dynamic dark energy or modifications to the laws of general relativity. Measuring the growth rate of $f\sigma_8$ structures through observations of the velocity field of galaxies is an effective way of detecting possible deviations from the standard model and discriminating between dynamical dark energy and modified gravity. In this thesis, I study the use of distance indicators such as type Ia supernovae to measure $f\sigma_8$. My work focuses on developing a simulation of the ZTF survey to study the biases and systematics of $f\sigma_8$ analysis. This analysis relies on the use of a velocity estimator that uses measurements of a distance modulus of an SN Ia and the *redshift* of the associated host galaxy. The velocities are then used in the maximization of a likelihood function with $f\sigma_8$ as a free parameter. A simulation of 27 ZTF mocks, each one equivalent to six years of survey, shows that the magnitude limit for spectroscopic classification of SNe Ia leads to a selection bias after a *redshift* $z \sim 0.06$. This bias is reflected in $f\sigma_8$. Limiting the data with a cutoff at $z \sim 0.06$ yields an unbiased measure of $f\sigma_8$ with an accuracy of 19%. This result is already comparable to those obtained with low-redshift galaxies. In addition, the use of a perfect bias correction to include SNe Ia at higher *redshifts* does not significantly reduce the error on $f\sigma_8$.

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Introduction

The standard model of cosmology assumes that gravity is described at all scales by general relativity (GR) and that its content is dominated by two exotic components, cold dark matter and a dark energy component described by a cosmological constant Λ . The exact nature and physical interpretation of this constant remain unknown. To explain it, some models propose replacing it with a new field. Other models modify the laws of general relativity in order to explain the observations without resorting to dark energy. Although these different models are similar in their description of the expansion of the Universe, they can be distinguished through the observation of large-scale matter structures. The growth rate of structures $f\sigma_8$ is a parameter linked to the evolution of large-scale structures. The most common measurements of this parameter are obtained through the observation of redshift-space distortions (RSD). These distortions arise because the velocity of a galaxy modifies the observed *redshift*, which distorts the estimated density field. The two-point correlation function, normally isotropic, ends up compressed along the line of sight. Measuring the anisotropy of this correlation function makes it possible to measure $f\sigma_8$.

Another way to measure $f\sigma_8$ is to directly estimate velocities. Estimating velocities requires measuring the *redshift* of a galaxy as well as its distance. Until now, the distance indicators used for such a measurement were galaxies, through the use of the Tully-Fisher relation and the Fundamental Plane. These indicators are far less precise than type Ia supernovae. However, the statistics of SNe Ia as well as their distribution across the sky did not make them competitive for this measurement. New SNe Ia surveys are now changing the situation, with the ZTF survey and Rubin-LSST, which will make it possible to measure tens of thousands of SNe Ia in the nearby Universe.

In this thesis we focus on the use of SNe Ia for a measurement of the growth rate of structures $f\sigma_8$. These standard candles are the subject of renewed interest in cosmology. Indeed, current (ZTF), upcoming (Rubin), and future (Roman) surveys are revolutionizing the statistics of SNe Ia and thus opening the door to the analysis of large-scale structures. This thesis work focuses on the observations of SNe Ia from the ZTF survey. To prepare the analysis of these data, we produced realistic simulations of the light curves of ZTF SNe Ia, including selection effects and instrumental noise, and we carried out the analysis required to derive $f\sigma_8$, based on the maximum likelihood method (Johnson et al. 2014 ; Howlett et al. 2017a).

Chapter 1 begins with a review of the theoretical foundations and the standard model of cosmology. It also presents a brief review of the tensions that this model still exhibits and of the possible solutions that motivate the research and analysis proposed in this thesis work. Chapter 2 introduces the physics of large-scale structures. In

particular, we focus on the velocity field, the cosmological parameters related to it, and its effects on our observations. Chapter 3 discusses the observation of type Ia supernovae and their use as standard candles and tracers of the velocity field in the context of cosmological analyses. Chapter 4 presents the ZTF photometric survey in which this thesis work took place. Chapter 5 presents the work on the simulation of ZTF observations. It also describes the analysis of the Hubble diagram of the simulated data and the estimation of peculiar velocities from it. Finally, Chapter 6 presents the method used to estimate $f\sigma_8$, the main results obtained, as well as a few variations of the analysis.

1 Cosmology

Etymologically, cosmology refers to the science that seeks to understand the cosmos. In practice, it aims to study, through observations, the origin, dynamics, and future of our Universe. For a long time, humankind could only rely on its eyes to observe the sky, and the interpretation of what was seen was shaped by culture. The myths describing the creation of the Universe are referred to as cosmogonies. We know that astrometry was already being practiced 3000 years ago, in Mesopotamia, by the Babylonians, for the religious reasons mentioned above, but also for navigation. During ancient Greece, the heliocentric system was already known, and the astronomer Aristarchus of Samos made the first estimates of the Earth-Moon and Earth-Sun distances. Astrometric techniques and instruments continued to develop in the Middle East and Asia during the European Middle Ages. During the Renaissance, the study of the sciences, and in particular celestial mechanics, developed further. At the beginning of the 16^e century, Copernicus proposed his heliocentric model. Initially rejected by the Church, this model became established during the 17^e century, thanks to technical progress in observation—for example, Galileo's telescope—but also thanks to advances in the description of the mechanics of celestial bodies, with Kepler's laws, followed by Newton's laws of gravitation. It was during the 20^e century, with the theory of special and then general relativity, that cosmology developed and established itself as the modern field of research we know today.

In this chapter, we propose to summarize the main aspects of cosmology. In Sec. [1.1](#), we present the theory and equations that allow us to describe our Universe. In Sec. [1.2](#), we describe the current status of the standard model of cosmology. Finally, in Sec. [1.3](#) we present some of the limitations of this model as well as its possible extensions.

1.1 Development of standard cosmology

At the beginning of the 20th century, physics underwent several conceptual revolutions. This paradigm shift manifested itself through the development of special relativity, which challenged the concept of time as an absolute quantity, and of quantum mechanics, which introduced a non-deterministic character into physics at the scale of particles. Subsequently, general relativity proposed a new theory of gravity based on a dynamical description of spacetime. The construction of these theories resulted from a dialogue between theoretical developments and experimental observations, the latter becoming increasingly precise thanks to technological progress. This interplay between theory and observations enabled advances in the understanding of physical phenomena at both small and large scales, and laid the foundations of the

two great models of modern physics: the standard models of particle physics and of cosmology. It is the latter that we will focus on here. In this section we present the standard model of cosmology, from its foundation in general relativity to its current limitations. This section is based largely on Peebles 1993 and Dodelson and Schmidt 2020.

1.1.1 Einstein's General Relativity

Special relativity

At the end of the 19^e century, the observation that light has a speed independent of the reference frame made the theories of electromagnetism and Newtonian mechanics incompatible. In 1905, Einstein reconciled the two theories by applying the principle of relativity, which states that "The laws of physics are the same in all inertial reference frames" (Einstein 1905). Applying this principle to the laws of electromagnetism leads to major conceptual changes: the speed of light is the same in all inertial reference frames, and consequently, the measurement of time is not invariant between these frames. The Galilean transformations, between two inertial frames, of the spatial coordinates (x, y, z) are then replaced by the Lorentz transformations of the spacetime coordinates (t, x, y, z) . Special relativity holds a special place in modern physics, since any valid theory is considered to be required to be compatible with it. This is in particular the case for the theory of general relativity, which Einstein would go on to develop, and on which the current model of cosmology is based.

The equivalence principle

Einstein's equivalence principle is an extension of the weak equivalence principle. The latter proposes that the inertial mass of objects is identical to their gravitational mass. Its consequence is the result already known to Galileo, and present in the Newtonian description of gravity, that the falling speed of objects is independent of their mass. Einstein extended this by proposing that, given the validity of the weak equivalence principle, an isolated observer cannot locally distinguish whether they are subject to a gravitational field or to a constant acceleration. This observation is the first step toward the concept of gravity as a geometric consequence of the shape of spacetime, rather than as a force.

Einstein's equation

The idea of giving spacetime a shape is formalized in mathematical terms using differential geometry: spacetime is a so-called Riemannian manifold and may be locally curved. To study it, spacetime is endowed with a metric $g_{\mu\nu}$, which makes it possible to define the notion of distance in spacetime. The curvature of spacetime is characterized by the Riemann tensor

$$R^\kappa_{\lambda\mu\nu} = \partial_\mu \Gamma^\kappa_{\lambda\nu} - \partial_\nu \Gamma^\kappa_{\lambda\mu} + \Gamma^\eta_{\nu\lambda} \Gamma^\kappa_{\mu\eta} - \Gamma^\eta_{\mu\lambda} \Gamma^\kappa_{\nu\eta}. \quad (1.1)$$

1 Cosmology – 1.1 Development of standard cosmology

where $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$ and the $\Gamma_{\mu\nu}^\eta$ are the Christoffel symbols defined by

$$\Gamma_{\mu\nu}^\eta = \frac{1}{2} g^{\eta\lambda} (g_{\mu\lambda} + g_{\nu\lambda} - \partial_\lambda g_{\mu\nu}). \quad (1.2)$$

The Christoffel symbols play an important role in general relativity: they make it possible to define the covariant derivative of tensors on spacetime

$$\nabla_\eta T_{\nu_1 \dots \nu_q}^{\mu_1 \dots \mu_p} = \partial_\eta T_{\nu_1 \dots \nu_q}^{\mu_1 \dots \mu_p} \quad (1.3)$$

$$+ \Gamma_{\eta\lambda}^{\mu_1} T_{\nu_1 \dots \nu_q}^{\lambda \dots \mu_p} + \dots + \Gamma_{\eta\lambda}^{\mu_p} T_{\nu_1 \dots \nu_q}^{\mu_1 \dots \lambda} - \Gamma_{\eta\nu_1}^\lambda T_{\lambda \dots \nu_q}^{\mu_1 \dots \mu_p} - \dots - \Gamma_{\eta\nu_q}^\lambda T_{\nu_1 \dots \lambda}^{\mu_1 \dots \mu_p} \quad (1.4)$$

From the Riemann tensor, we construct the Ricci tensor

$$R_{\mu\nu} = R^\kappa_{\mu\kappa\nu}, \quad (1.5)$$

and finally the scalar curvature

$$\mathcal{R} = R^\mu_\mu \quad (1.6)$$

The mathematical objects that we have just defined are at the heart of Einstein's equation. This equation, published in 1916 (Einstein 1916), relates the curvature of spacetime to its content, described by the energy-momentum tensor $T_{\mu\nu}$. It is written

$$R_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (1.7)$$

where we use the natural unit system $c = \hbar = 1$, G is the gravitational constant, and Λ is the cosmological constant. This constant is an addition permitted within the theory; we will see that it plays a central role today in understanding the accelerating expansion of the Universe. One can also note that the term $8\pi G$ makes it possible to recover Newton's equations in the limit of a weak gravitational field. Part of the left-hand term is generally grouped into a tensor called the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu}. \quad (1.8)$$

Einstein's equation is constructed so as to respect the local conservation of the energy-momentum tensor imposed by Noether's theorem

$$\nabla_\mu G^\mu_\nu = \Lambda \nabla_\mu g^\mu_\nu = \nabla_\mu T^\mu_\nu = 0. \quad (1.9)$$

The authorship of Einstein's equation has been the subject of debate. Indeed, Hilbert proposed, at the same time, an action whose variational principle with respect to a variation of the metric $\delta g_{\mu\nu}$ leads to the same equation (Eq. 1.7) (Hilbert 1915). This

action, known as the "Einstein-Hilbert" action, is written

$$S_{\text{EH}} = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} (\mathcal{R} - 2\Lambda) + \mathcal{L}_m \right], \quad (1.10)$$

where g is the determinant of the metric and $\mathcal{L}_m$ is the Lagrangian of the fields considered in the theory (matter, radiation, ...).

1.1.2 Solving Einstein's equation for the Universe

Cosmological principle

Einstein's equation is not easy to solve in the general case. To restrict the possible solutions, Einstein proposed what is called the cosmological principle, which relies on the hypothesis that there is no privileged observer in the Universe. This principle can be stated as follows: "On sufficiently large scales, the Universe appears homogeneous and isotropic." Although, at the time it was formulated, this principle could not be verified for lack of technical means, it was quickly accepted. One of the modern arguments in favor of this principle is the observation by *Planck* of CMB anisotropies of order 10^{-5} K.

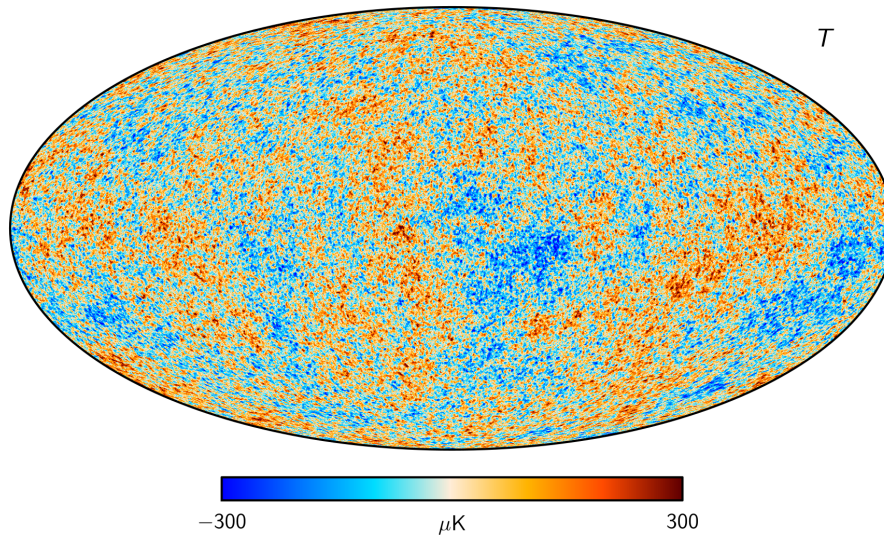


Figure 1.1 – Map of the CMB temperature anisotropies produced by the *Planck 2018* analysis. The small amplitude of the anisotropies, of order μK , is an argument in favor of the cosmological principle. Credits: esa.int

A metric to describe the Universe

The general expression for the metric of a Euclidean universe is given by

$$ds^2 = -dt^2 + a_{ij}(t, \mathbf{x}) dx^i dx^j \quad (1.11)$$

1 Cosmology – 1.1 Development of standard cosmology

A choice of orthogonal coordinates leads to $a_{ij} = 0$ for $i \neq j$. We now apply the cosmological principle. Homogeneity implies that the terms $a_{ii}(t, \mathbf{x})$ must be independent of position: $a_{ii}(t, \mathbf{x}) = a_{ii}(t)$. Isotropy requires the metric to be the same in all directions, $\forall i \ a_{ii}(t) = a(t)$. The metric can then be written

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (1.12)$$

where $a(t)$ is chosen as a dimensionless parameter such that $a(t_0) = 1$ for a chosen time t_0 . The parameter $a(t)$ is called the scale factor; its time variation describes the expansion of the Universe. This metric, valid in the Euclidean case, can be generalized to three universe geometries by introducing the Gaussian curvature of the Universe k such that

$$k = \frac{1}{R_0^2}, \quad (1.13)$$

where R_0 is the radius of curvature and has the dimension of a distance. The sign of k then determines the geometry of the Universe

$$\begin{cases} k > 0 & \text{Closed Universe} \\ k = 0 & \text{Flat Universe} \\ k < 0 & \text{Open Universe} \end{cases} \quad (1.14)$$

The so-called Friedmann-Lemaitre-Robertson-Walker (FLRW) metric is then written

$$ds^2 = -dt^2 + a^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right\}. \quad (1.15)$$

The form of the FLRW metric in Eq. 1.15 is pathological for the coordinate r at $r = R_0$ in the case of a closed Universe $k > 0$. This problem is avoided by performing the coordinate change $r \rightarrow \chi$ with $r = S_k(\chi)$, where the function $S_k(\chi)$ is defined by

$$S_k(\chi) = \begin{cases} R_0 \sin\left(\frac{\chi}{R_0}\right) & k > 0 \\ \chi & k = 0 \\ R_0 \sinh\left(\frac{\chi}{R_0}\right) & k < 0 \end{cases} \quad (1.16)$$

The metric then takes the form

$$ds^2 = -dt^2 + a^2(t) \{d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)\}. \quad (1.17)$$

The content of the Universe: the energy-momentum tensor

The previous section presented the FLRW metric, which describes the shape of the spacetime of the Universe. We have thus described the right-hand side of Einstein's equation (Eq. 1.7). We now need to describe the left-hand side of the equation: the content of the Universe. This content is represented, in the theory of general relativity, by the energy-momentum tensor T^μ_ν .

The cosmological principle imposes, as it did for the FLRW metric, the form of the energy-momentum tensor

$$T^\mu{}_\nu = \begin{pmatrix} -\rho(t) & & & \\ & P(t) & & \\ & & P(t) & \\ & & & P(t) \end{pmatrix}, \quad (1.18)$$

where ρ is an energy density and P a pressure. The energy-momentum tensor of Eq. 1.18 corresponds to that of a perfect fluid at rest, i.e., with a four-velocity $u^\mu = (-1, 0, 0, 0)$. This is sometimes written

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu + p g^{\mu\nu}. \quad (1.19)$$

If we apply the covariant derivative defined by Eq. 1.4 to the energy-momentum tensor of Eq. 1.18 to obtain the local conservation equation (Eq. 1.9), we find that the only component that is not identically zero is

$$\nabla_\mu T^\mu{}_0 = \frac{\partial \rho}{\partial t} + 3 \frac{\dot{a}}{a} (\rho + P) = 0. \quad (1.20)$$

The following relation is called the equation of state

$$P = w\rho, \quad (1.21)$$

where w is called the equation-of-state parameter. Eq. 1.20 can be rewritten as

$$\dot{\rho} = -3\rho(1+w) \frac{\dot{a}}{a}. \quad (1.22)$$

This last equation can be solved in the form

$$\frac{d\rho}{\rho} = -3(1+w) \frac{da}{a}. \quad (1.23)$$

The general solution to such an equation is

$$\rho(a) = \rho_0 \exp \left[-3 \int_{a_0}^a (1+w) \frac{da'}{a'} \right] \quad (1.24)$$

$$\stackrel{w=\text{cst}}{=} \rho_0 \left(\frac{a}{a_0} \right)^{-3(1+w)}, \quad (1.25)$$

where ρ_0 and a_0 are, respectively, the energy density and the scale factor at t_0 .

The equations we have just derived hold in general. We now need to determine the equation-of-state parameter of the species contained in the Universe. From the point of view of statistical physics, the distribution function $f_i(\mathbf{x}, \mathbf{p}, t)$ of a species counts

the particles in each infinitesimal element $d^3\mathbf{p}$ of phase space $(\mathbf{x}, \mathbf{p})$. Knowledge of $f_i(\mathbf{x}, \mathbf{p}, t)$ thus makes it possible to obtain the energy density through the integral

$$\rho_i(\mathbf{x}, t) = g_i \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_i(\mathbf{x}, \mathbf{p}, t) E_i(\mathbf{p}), \quad (1.26)$$

where g_i is the degeneracy of the species and E_i the energy. Similarly, the pressure is given by

$$P_i(\mathbf{x}, t) = g_i \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_i(\mathbf{x}, \mathbf{p}, t) \frac{p^2}{3E_i(\mathbf{p})}. \quad (1.27)$$

In general, to determine the evolution of the function f , one must solve the Boltzmann equation

$$\frac{df}{dt} = C[f], \quad (1.28)$$

where $\frac{d}{dt} = \partial_t + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}}$ and $C[f]$ is the collision term corresponding to the interactions. This approach is the one used by Boltzmann-equation solver codes such as CAMB¹ and CLASS². It is useful for studying structure formation, where one must consider the inhomogeneities in the distribution of the different species of the Universe. In the context of the overall evolution of the Universe, the assumptions of isotropy and homogeneity have the consequence of removing the spatial dependence of the distribution function, so that $f_i(\mathbf{x}, \mathbf{p}, t) \equiv f_i(p, t)$, and hence $\rho_i(\mathbf{x}, \mathbf{p}, t) \equiv \rho_i(p, t)$ and $P_i(\mathbf{x}, \mathbf{p}, t) \equiv P_i(p, t)$. Moreover, if the species under consideration is assumed to be in equilibrium at temperature T , its distribution function follows either Bose-Einstein statistics (for bosons) or Fermi-Dirac statistics (for fermions). We then have the solution for f

$$f_{\text{FD/BE}}(E) = \frac{1}{\exp[(E - \mu)/T] \pm 1}, \quad (1.29)$$

where $+1$ is used for Fermi-Dirac, -1 for Bose-Einstein, E is the energy $E = \sqrt{p^2 + m^2}$, and μ is the chemical potential of the species. We now apply these equations to the two species generally considered, non-relativistic matter and photons.

For non-relativistic matter, i.e., when the rest-mass energy dominates over the thermal energy $m \gg T$ and the energy can be approximated by $E \simeq m \left(1 + \frac{p^2}{2m^2}\right)$, the distribution function of Eq. 1.29 becomes the same for fermions and bosons

$$f_{nr}(T) \simeq e^{-(m-\mu)/T} e^{-p^2/2mT}, \quad (1.30)$$

1. <https://camb.info/>

2. https://lesgourg.github.io/class_public/class.html

1 Cosmology – 1.1 Development of standard cosmology

Computing the density (Eq. 1.26) and the pressure (Eq. 1.27) gives us

$$\begin{aligned}\rho_{nr}(T) &\simeq g \frac{m^{5/2}}{2\pi^{3/2}} T^{3/2} e^{(\mu-m)/T} \\ P_{nr}(T) &\simeq g \frac{m^{3/2}}{2\sqrt{2}\pi^{3/2}} T^{5/2} e^{(\mu-m)/T}\end{aligned}\quad (1.31)$$

and the equation of state is

$$w_{nr} = \frac{P_{nr}}{\rho_{nr}} \simeq \frac{T}{\sqrt{2}m} \xrightarrow{m \gg T} 0 \quad (1.32)$$

We therefore consider that non-relativistic matter has a vanishing equation-of-state parameter w . According to Eq. 1.25, this leads to a solution for the evolution of the energy density

$$\rho_{nr} = \rho_{nr,0} a^{-3}. \quad (1.33)$$

This solution is the one considered for known matter, called baryonic matter.

For relativistic species, such as photons or neutrinos, we have $T \gg m, \mu$ (for photons $\mu = 0$), and the calculations of the density and pressure lead to

$$\begin{aligned}\rho_r(T) &= g_i \frac{\zeta(3)\pi^2}{30} T^4 \begin{cases} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{cases} \\ P_r(T) &= \frac{1}{3} \rho_r(T)\end{aligned}\quad (1.34)$$

We immediately deduce that the equation of state for relativistic species is

$$w_r = \frac{1}{3}. \quad (1.35)$$

Using this result in Eq. 1.25, we obtain the density evolution equation for relativistic species

$$\rho_r = \rho_{r,0} a^{-4}. \quad (1.36)$$

To obtain the general energy-momentum tensor of the Universe, we neglect all interactions between the species present. The energy-momentum tensor of our Universe can then be expressed as the sum of the energy-momentum tensors of all the constituents i of our Universe

$$T^\mu_\nu = \sum_i [T_i]^\mu_\nu. \quad (1.37)$$

Friedmann's equations

By inserting the metric of Eq. 1.17 into the left-hand side of Einstein's equation (Eq. 1.7) and the expression for the energy-momentum tensor of Eq. 1.37 into the

right-hand side, we obtain Friedmann's two equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i + \frac{\Lambda}{3} - \frac{k}{a^2} \quad (1.38)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i) + \frac{\Lambda}{3}. \quad (1.39)$$

We can note that combining Eqs. 1.38 and 1.39 recovers Eq. 1.22. Another observation concerning the cosmological constant Λ is that, in the second Friedmann equation (Eq. 1.39), it is the only term on the right-hand side that can be positive, and can thus counterbalance the deceleration of the expansion of the Universe ($\ddot{a} < 0$) caused by gravity. Einstein introduced it to make use of this property and render his model of the Universe static.

Cosmological parameters

The Hubble parameter is the quantity on the left-hand side of Eq. 1.38

$$H = \frac{\dot{a}}{a}, \quad (1.40)$$

and H_0 denotes the Hubble constant, the evaluation of H at our current epoch $t = t_0$. The parameter H has the dimension of inverse time but is often expressed as an "expansion speed" in $\text{km.s}^{-1}.\text{Mpc}^{-1}$. The cosmological-constant term in the first equation (Eq. 1.38) can be reformulated to express it as an energy density

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}. \quad (1.41)$$

To apply this formulation to the second equation, we solve

$$\frac{\Lambda}{3} = -\frac{4\pi G}{3} \rho_\Lambda (1 + 3w_\Lambda) \quad (1.42)$$

We then find the value of the equation-of-state parameter of the cosmological constant

$$w_\Lambda = -1. \quad (1.43)$$

This equation-of-state parameter corresponds to that of a perfect fluid with negative pressure.

The same operation can be performed with the curvature term, by setting

$$\rho_k = -\frac{3}{8\pi G} \frac{k}{a^2}. \quad (1.44)$$

Friedmann's equations can then be expressed as

$$H^2 = \frac{8\pi G}{3} \left(\rho_\Lambda + \rho_k + \sum_i \rho_i \right) \quad (1.45)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho_\Lambda(1 + 3w_\Lambda) + \sum_i \rho_i(1 + 3w_i) \right). \quad (1.46)$$

The critical density ρ_{crit} is defined as the density for which the Universe is Euclidean, i.e., the total density $\rho = \rho_\Lambda + \sum_i \rho_i$ such that $k = \rho_k = 0$

$$\rho_{\text{crit}}(a) = \frac{3H^2(a)}{8\pi G} \quad (1.47)$$

We can normalize the densities by the critical density at t_0 to obtain the dimensionless density parameters

$$\Omega_i(a) = \frac{\rho_i(a)}{\rho_{\text{crit},0}}. \quad (1.48)$$

The first Friedmann equation can then be written

$$H^2(a) = H_0^2 \left(\Omega_\Lambda + \Omega_k(a) + \sum_i \Omega_i(a) \right) \quad (1.49)$$

This expression can be given when we consider the (non-relativistic) matter density Ω_m and the radiation density (photons + neutrinos) Ω_r . Making the dependence on a explicit, we obtain

$$H^2(a) = H_0^2 (\Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{\Lambda,0} + \Omega_{k,0} a^{-2}), \quad (1.50)$$

where the $\Omega_{i,0}$ are the density parameters at t_0 , and which, by construction, forms the closure relation

$$\sum_{i \neq k} \Omega_{i,0} = 1 - \Omega_{k,0}. \quad (1.51)$$

There is another convention in which one normalizes by the critical density at all times $\rho_{\text{crit}}(a)$ instead of $\rho_{\text{crit},0}$ fixed at t_0 in Eq. 1.48. The closure relation then holds at all times t . In the convention used here, we have

$$\sum_{i \neq k} \Omega_i(a) E^{-2}(a) = 1 - \Omega_k(a) E^{-2}(a), \quad (1.52)$$

where $E(a)$ is the dimensionless function such that $H(a) = H_0 E(a)$. Fig. 1.2 shows the evolution of the density parameters as measured by Planck Collaboration et al. 2020. We can see that the Universe is leaving the matter-dominated era and entering the dark-energy-dominated era. In Fig. 1.2 we have also added the neutrino energy density Ω_ν . Neutrinos behave, in the early Universe, as relativistic particles at first. They then cool as the Universe expands and transition into a non-relativistic regime,

where their energy is dominated by their rest-mass energy.

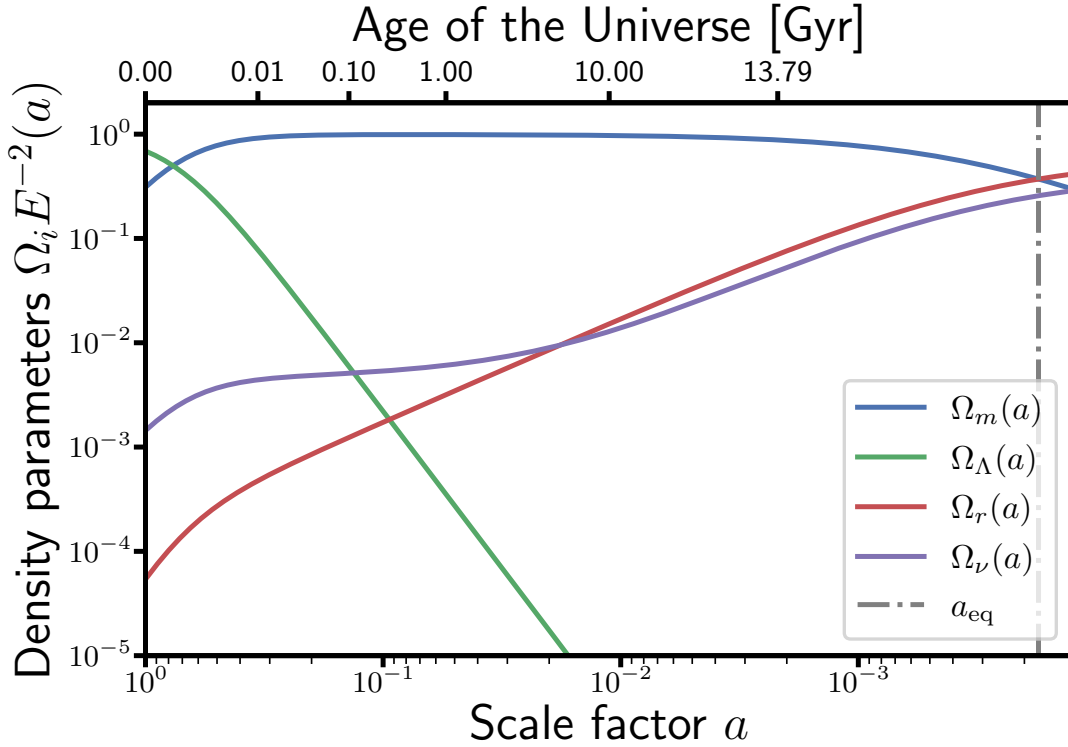


Figure 1.2 – Evolution of the density parameters with the values measured by Planck Collaboration et al. 2020. The (dark and baryonic) matter density parameter Ω_m is shown in blue, the dark-energy density parameter Ω_Λ is shown in green, the radiation-energy density parameter Ω_r is shown in red, and the neutrino energy density parameter is shown in purple. The curvature parameter Ω_k is zero.

1.1.3 Redshift and distance measurements

Hubble-Lemaître law

In 1929, Edwin Hubble observed that the spectrum of nearby galaxies is shifted as a function of their distance. This shift was then interpreted as the consequence of a Doppler effect linked to a radial recession velocity. Using a distance measurement based on Cepheids and the angular diameter of galaxies (Fig. 1.3), he established the law (Hubble 1929)

$$v = cz = H_0 D, \quad (1.53)$$

where v is the recession velocity of the galaxies, D their distance, and z the *redshift* (shift toward the red) which quantifies the ratio between the observed wavelength λ_o

and the emitted wavelength λ_s

$$1 + z = \frac{\lambda_o}{\lambda_s}. \quad (1.54)$$

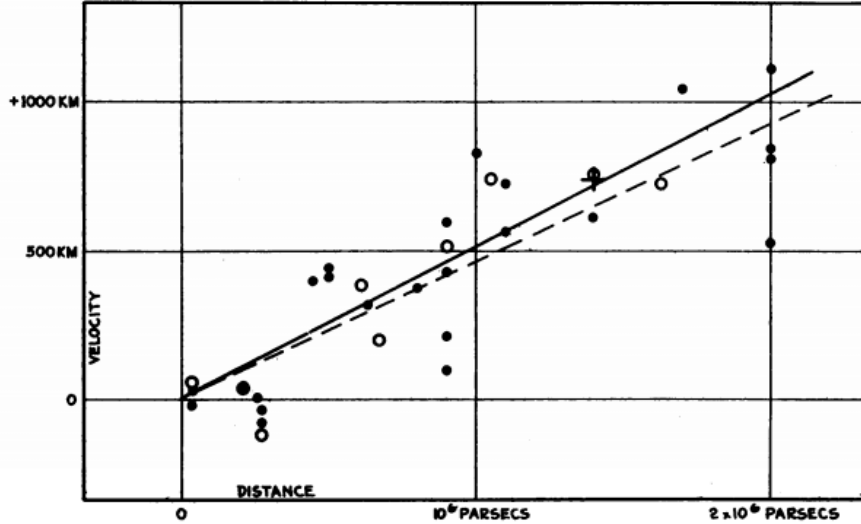


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Figure 1.3 – Relation between the recession velocity of galaxies and their distances Hubble 1929

This result had already been predicted as a consequence of general relativity applied to Universe models by Lemaître in 1927 (Lemaître 1927). Indeed, this observation is due to the expansion of the Universe and can be related to the scale factor a .

The trajectories of photons through spacetime are described by the null geodesics $ds^2 = 0$ of the FLRW metric of Eq. 1.17. Along the line of sight ($d\theta = d\phi = 0$), we thus have the relation

$$d\chi = \pm \frac{dt}{a(t)}. \quad (1.55)$$

We now consider a source that emits a light signal of wavelength λ_s ; the period of the signal in the source's reference frame is thus $\delta t_s = \frac{c}{\lambda_s}$. A first wavefront is emitted at t_s and received at t_0 , so that

$$\int_{\chi}^0 d\chi' = - \int_{t_s}^{t_0} \frac{dt}{a(t)}. \quad (1.56)$$

The second wavefront is emitted at $t_s + \delta t_s$ and received at $t_0 + \delta t_o$, so we have a relation identical to the previous one

$$\int_{\chi}^0 d\chi' = - \int_{t_s + \delta t_s}^{t_0 + \delta t_o} \frac{dt}{a(t)}. \quad (1.57)$$

We then have the equality

$$\int_{t_s}^{t_0} \frac{dt}{a(t)} = \int_{t_s+\delta t_s}^{t_0+\delta t_o} \frac{dt}{a(t)} \quad (1.58)$$

$$= \int_{t_s}^{t_0} \frac{dt}{a(t)} + \int_{t_0}^{t_0+\delta t_o} \frac{dt}{a(t)} - \int_{t_s}^{t_s+\delta t_s} \frac{dt}{a(t)}, \quad (1.59)$$

which gives us the relation

$$\int_{t_0}^{t_0+\delta t_o} \frac{dt}{a(t)} = \int_{t_s}^{t_s+\delta t_s} \frac{dt}{a(t)}. \quad (1.60)$$

We can consider a to be constant between t_0 and $t_0 + \delta t_o$ and between t_s and $t_s + \delta t_s$. We finally obtain the relation between the time intervals δt_o and δt_s

$$\frac{\delta t_o}{\delta t_s} = \frac{a(t_0)}{a(t_s)}. \quad (1.61)$$

Since the wavelength measured by the observer is $\lambda_o = c\delta t_o$, it is given by

$$\frac{\lambda_o}{\lambda_s} = \frac{a(t_0)}{a(t_s)} = 1 + z. \quad (1.62)$$

Note that Eq. 1.62 is only valid within the framework where the assumptions of the FLRW metric hold. That is, the source and the observer are assumed to have no velocity relative to the comoving frame, and are separated by scales large enough for the Universe to be considered isotropic and homogeneous there. We will see later that the relation shown by Hubble is an approximation that is only valid at small distances. In cosmology, it is possible to define several types of distances.

Comoving distance

The comoving distance χ corresponds to the distance that can be measured between two points if we freeze the expansion of the Universe today, i.e., it is corrected for this expansion. It can be defined by measuring the variation of the coordinate χ for a photon emitted by a source at a time t_e and reaching us today at t_0 . This case is identical to that of Eq. 1.56. The definition of H in Eq. 1.40 gives the relation $da = aHdt$, allowing us to make the change of variable $t \rightarrow a$

$$\chi(a) = - \int_a^1 \frac{da'}{a'^2 H(a')} = \int_1^a \frac{da'}{a'^2 H(a')}. \quad (1.63)$$

This distance can also be written in terms of redshift z . We perform the change of variable $a \rightarrow z$ using Eq. (1.62). We then have the relation between da and dz

$$da = -a^2 dz, \quad (1.64)$$

and the comoving distance is written as a function of redshift

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}. \quad (1.65)$$

Transverse comoving distance

The transverse comoving distance D_A is defined such that the comoving distance $D_\perp$ between two objects located at the same distance χ from the observer and separated by an infinitesimal angle $\delta\theta$ is given by $D_\perp = D_A \delta\theta$. Using the metric of Eq. 1.17 in the case described, we deduce that D_A is given by

$$D_A(z) = S_k(\chi(z)) \quad (1.66)$$

Angular diameter distance

The angular diameter distance d_A is defined in a manner similar to the transverse comoving distance. Unlike the latter, it is not measured in a universe fixed at t_0 . It is defined by the ratio between the proper distance $d = aD_\perp = \frac{D_\perp}{1+z}$ between the two objects at the moment the light we receive today was emitted, and their angular separation $\delta\theta$. We thus have

$$d_A(z) = \frac{1}{1+z} \frac{D_\perp}{\delta\theta} = \frac{D_A(z)}{1+z} = \frac{S_k(\chi(z))}{1+z} \quad (1.67)$$

Luminosity distance

The luminosity distance d_L is the distance at which a source of luminosity L emitting isotropically would need to be located for us to observe the flux F :

$$F = \frac{L}{4\pi d_L^2}. \quad (1.68)$$

To simplify, we can imagine a source that emits, isotropically and at a constant rate $\dot{N}$, photons of fixed energy $E = \frac{hc}{\lambda}$. The luminosity of the source is

$$L = \dot{N} \frac{hc}{\lambda_s}. \quad (1.69)$$

The photons that reach us from this source are emitted within a solid angle Ω_s , so we can write their number as

$$N = \dot{N} \delta t_s \frac{\Omega_s}{4\pi} = L \delta t_s \frac{\lambda_s \Omega_s}{hc 4\pi}. \quad (1.70)$$

1 Cosmology – 1.1 Development of standard cosmology

We collect these same photons over a surface A_o , during a time interval δt_o , and their observed wavelength is λ_o . The measured flux is

$$F = \frac{N}{\delta t_o A_o} \frac{hc}{\lambda_o} = \frac{L}{4\pi} \frac{\lambda_s}{\lambda_o} \frac{\delta t_s}{\delta t_o} \frac{\Omega_s}{A_o}. \quad (1.71)$$

By identifying the flux F from equations (1.68) and (1.71), we can give the expression for the luminosity distance

$$d_L = \sqrt{\frac{\delta t_o}{\delta t_s} \frac{\lambda_o}{\lambda_s} \frac{A_o}{\Omega_s}}. \quad (1.72)$$

We have seen previously that $\delta t_o = (1+z)\delta t_s$ and $\lambda_o = (1+z)\lambda_s$. We now apply the same reasoning as for the transverse comoving distance, but with the roles of the source and observer reversed. The validity of this reasoning is guaranteed by Etherington's reciprocity theorem, which holds for any Riemannian space, independently of Einstein's equations (Ellis 2007). Thus, at the instant t_0 at which we receive the photons, the size $D_\perp$ subtended by an angle $\delta\theta$ as seen from the source is $D_\perp = S_k(\chi)\delta\theta$, where χ is the comoving distance between the source and the observer. Generalizing to the surface $A_o \equiv D_\perp^2$, we obtain $A_o = S_k^2(\chi)\Omega_s$. We finally obtain

$$d_L(z) = \sqrt{(1+z)^2 S_k^2(\chi(z))} = (1+z) S_k(\chi(z)). \quad (1.73)$$

Figure 1.4 shows the different distances for the values of the density parameters measured by the Planck collaboration (Planck Collaboration et al. 2020). We can see that at low *redshift* the different distances coincide. The redshift-distance relations are often summarized under the generic name of the "Hubble diagram." These relations are of primary importance for cosmology, as they depend on the density parameters describing the content of our Universe; thus, one can compare the Hubble diagram of a given cosmological model with that obtained from observations.

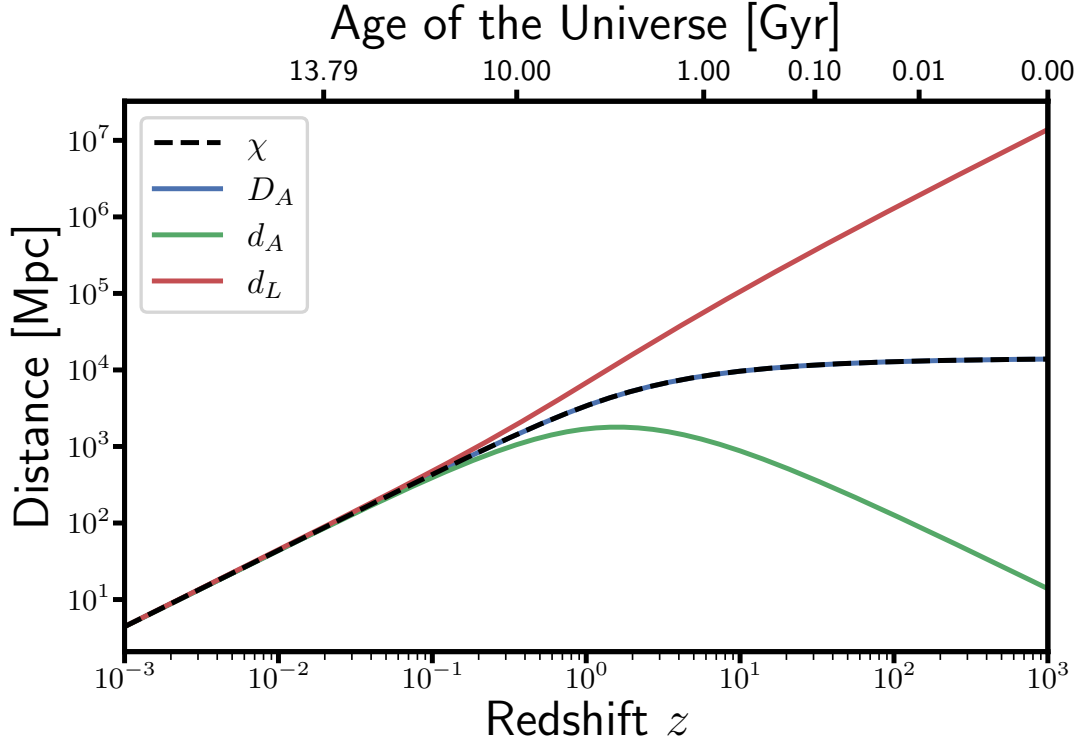


Figure 1.4 – Cosmological distances as a function of *redshift*. We use the parameters measured by Planck (Planck Collaboration et al. 2020), which correspond to a flat universe with $\Omega_{m,0} \approx 0.3$.

1.2 The Λ CDM concordance model

In the previous section, we presented the formalism of standard cosmology and how it follows from the theory of general relativity. However, the construction of the standard model of cosmology, the Λ CDM model, relies on the constraints that observations bring to the theory.

1.2.1 Big Bang and primordial nucleosynthesis

An extrapolation of the observational evidence for an expanding Universe leads to the conclusion that it must have been very hot and dense in its early stages. In this primordial universe, high-energy photons prevent the formation of atomic nuclei. The idea was then proposed that the synthesis of elements could occur once the plasma making up the primordial Universe began to cool due to the expansion (Gamow 1946; Alpher et al. 1948). Calculations based on the cosmological model and nuclear physics make it possible to predict the production of light nuclei, up to lithium ${}^7\text{Li}$ (Fig. 1.5).

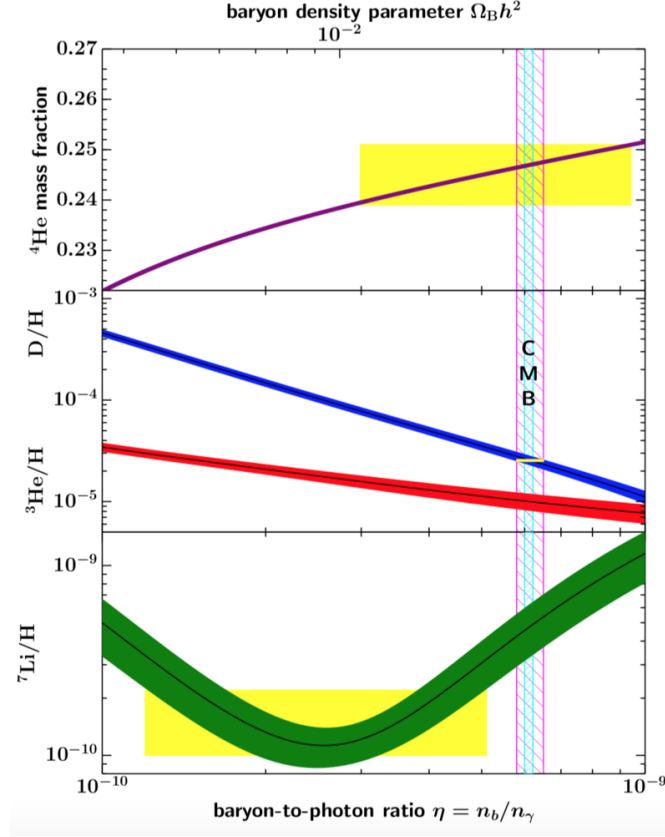


Figure 1.5 – Evolution of the abundances of the elements ^4H , D , ^3He , and ^7Li predicted by the primordial nucleosynthesis model as a function of the baryon-to-photon ratio. The bands represent the 95% confidence contours, the yellow bands are the observational limits, and the vertical band represents the 95% contour of the baryon-photon ratio measurement from the CMB. Credit: Workman and Others [2022](#)

Comparing observations with this theory provides constraints on the baryonic matter energy density Ω_b . The current constraints (Planck Collaboration et al. [2020](#)) are

$$\Omega_{b,0} h^2 = 0.02242 \pm 0.00014, \quad (1.74)$$

where h is the reduced Hubble parameter $h = H_0/100 \text{ km.s}^{-1}\text{Mpc}^{-1}$.

1.2.2 Cosmic Microwave Background

The plasma of the primordial Universe described in the previous section is opaque to photons. Indeed, in the earliest moments of the Universe, photons are highly energetic and prevent electrons and protons from remaining bound to form hydrogen through the reaction



The free electrons then scatter the photons via Thomson scattering, which prevents the latter from traveling freely. As the temperature decreases with the expansion, the energy of the photons is no longer sufficient to ionize hydrogen atoms, and the electrons recombine with the protons. The photons can then travel freely through our Universe. This collection of photons is called the cosmic microwave background (CMB). Its epoch of emission is measured at the so-called recombination *redshift* $z_* = 1089.80 \pm 0.21$ (Planck Collaboration et al. 2020).

The CMB was detected for the first time in 1965 by A. Penzias and R. Wilson (Penzias and Wilson 1965). This radiation is almost perfectly modeled by the blackbody law, and its temperature is measured precisely. In 1992, the measurement of the CMB monopole by COBE gave the constraint $T_{\text{CMB}} = 2.726 \pm 0.010$ (Mather et al. 1994); a combination of the various measurements available in 2009 refined this result to $T_{\text{CMB}} = 2.72548 \pm 0.00057$ (Fixsen 2009). CMB photons are the largest contribution to the radiation energy density; this density is obtained using Eq. 1.34, and the result is

$$\Omega_{\gamma,0} \simeq 5.38 \times 10^{-5}. \quad (1.76)$$

Although almost perfectly homogeneous, the CMB contains small variations related to the initial fluctuations of the metric $\mathcal{R}$. Their evolution will lead to the formation of the structures we know today. The spatial correlation between the $\mathcal{R}$ fluctuations is modeled as a Gaussian random field, i.e., a field whose information is encoded in Fourier space by the power spectrum $P_{\mathcal{R}}(k)$ (see Sec. 2.1.1). For the CMB, this power spectrum can generally be expressed as

$$\frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) \simeq A_s \left(\frac{k}{k_0} \right)^{n_s-1}, \quad (1.77)$$

where k_0 is a certain pivot scale, A_s is the amplitude of the perturbations, and n_s is the spectral index, which indicates the scale dependence of the power spectrum. The current measurements, for a pivot scale fixed at $k_0 = 0.05 \text{Mpc}^{-1}$ (Planck Collaboration et al. 2020), are

$$\ln(10^{10} A_s) = 3.047 \pm 0.014 \quad (1.78)$$

$$n_s = 0.9665 \pm 0.0038. \quad (1.79)$$

Since $n_s \simeq 1$, the power spectrum is almost independent of the scale k .

1.2.3 Dark Ages and reionization

After the emission of the CMB photons, while stars have not yet formed, the Universe contains no visible photons; this period is called the dark ages. The primordial fluctuations evolve and eventually form the structures within which stars are born through gravitational collapse. These stars begin to emit photons that ionize the interstellar medium, which had been neutral since recombination; this is reionization. The electrons in the ionized medium interact with the CMB photons through Thomson

scattering, which has observational consequences for the CMB. The probability of a photon interacting is $\propto e^{-\tau}$, where τ is the optical depth

$$\tau = \sigma_T \int_0^{z_*} dz \frac{n_e(z)}{(1+z)H(z)}, \quad (1.80)$$

with σ_T the Thomson cross section and $n_e(z)$ the mean density of free electrons at z . The current measurement (Planck Collaboration et al. 2020) is

$$\tau = 0.0561 \pm 0.0071 \quad (1.81)$$

1.2.4 Dark matter

In the 1930s, physicist Fritz Zwicky noticed a large discrepancy between the estimated mass of the Coma cluster, based on the luminosity and number of galaxies, and their velocities (Zwicky 1933; Zwicky 1937). He then proposed the existence of dark matter to account for the missing mass. Other observations, notably made by Vera Rubin on the measurement of galaxy rotation curves (Rubin and Ford 1970), would go on to support the existence of dark matter (Fig. 1.6).

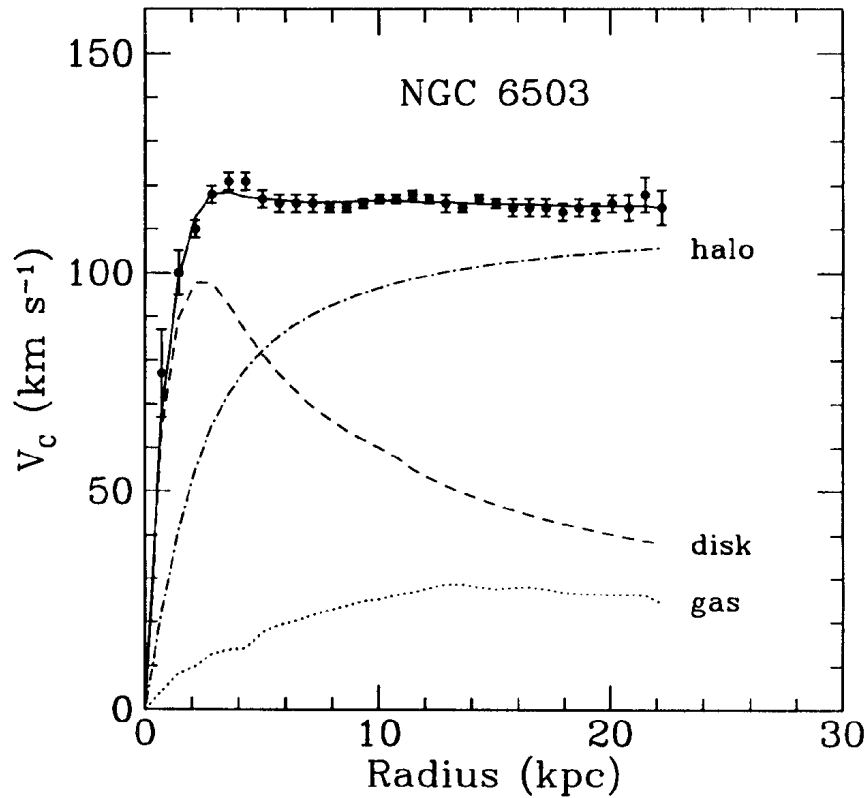


Figure 1.6 – Rotation velocity as a function of distance from the center of the galaxy NGC 6503. The *disk* and *gas* curves represent the contribution of baryonic matter to the mass; the *halo* curve represents the contribution of dark matter needed to explain the data. Credits: Freese [2017](#)

Dark matter is not explained today by the standard model of particle physics. Observations indicate that it must be massive and that it does not interact (or interacts very little) with baryonic matter, as shown by images of the *Bullet Cluster* (figure [1.7](#)). Moreover, the formation of galaxies, clusters, and superclusters suggests that this dark matter must be non-relativistic, i.e., cold (*Cold Dark Matter*).

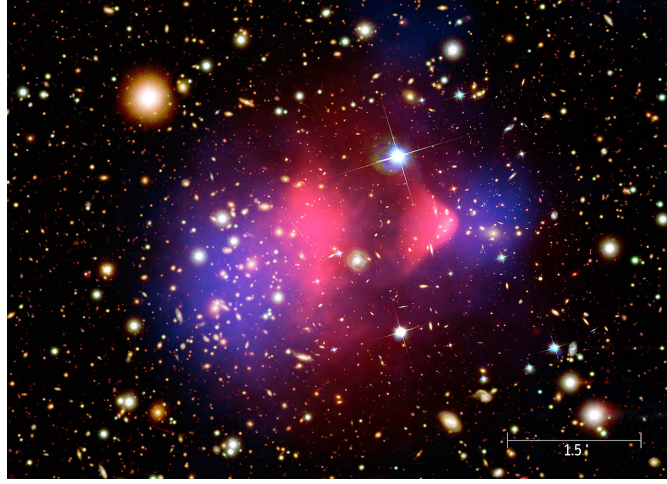


Figure 1.7 – *Bullet cluster*. Baryonic matter is traced using X-rays (in red); the mass distribution is measured using gravitational lensing (in blue). Credits: NASA

The best constraint on this parameter is (Planck Collaboration et al. 2020)

$$\Omega_{\text{cdm},0}h^2 = 0.11933 \pm 0.00091. \quad (1.82)$$

The dark and baryonic energy densities are combined into the matter energy density parameter

$$\Omega_m = \Omega_b + \Omega_{\text{cdm}}, \quad (1.83)$$

the current measurement of this parameter (Planck Collaboration et al. 2020) is

$$\Omega_{m,0} = 0.3111 \pm 0.0056 \quad (1.84)$$

1.2.5 Dark energy

In 1998 and 1999, two independent teams measured, using the distance-*redshift* relation of SNe Ia, that the expansion of the Universe is accelerating (Riess et al. 1998; Perlmutter et al. 1999). This observation rehabilitated the cosmological constant Λ , which had been abandoned by Einstein, since, as explained in Sec. 1.1.2, it makes it possible to obtain a model of an accelerating expanding Universe consistent with the observations. However, although the existence of the cosmological constant is permitted within the context of general relativity, it lacks a satisfactory physical explanation. For example, its interpretation as vacuum energy is incompatible with the result for the Planck energy density obtained in quantum field theory, $\rho_\Lambda = 10^{-120} \rho_{\text{planck}}$ (Carroll 2001). The current measurement of the dark energy density (Planck Collaboration et al. 2020) is

$$\Omega_{\Lambda,0} = 0.6889 \pm 0.0056. \quad (1.85)$$

To search the observations for a possible evolution of dark energy as a function of

redshift z (see Sec. 1.3.2), we use the CPL parametrization (Chevallier and Polarski 2000; Linder 2003) to describe its equation of state:

$$w_\Lambda = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{1+z}. \quad (1.86)$$

This equation of state leads to the expression for the dark energy density

$$\Omega_\Lambda = \Omega_{\Lambda,0}(1+z)^{3(1+w_0+w_a)} e^{w_a \frac{z}{1+z}}. \quad (1.87)$$

The current measurements do not appear to show a significant deviation from a constant dark energy density and remain compatible with $w_0 = -1$ and $w_a = 0$.

In summary, the standard model of cosmology, Λ CDM, relies on six parameters: Ω_b , Ω_{cdm} , H_0 , A_s , n_s , and τ . Note that Ω_Λ is not among these parameters, since it is constrained through the closure relation of Eq. 1.51. Indeed, the Λ CDM model is considered to have zero curvature, since it is measured to be very small, $\Omega_{k,0} = 0.0007 \pm 0.0019$ (Planck Collaboration et al. 2020). We thus have $\Omega_{\Lambda,0} = 1 - \Omega_{m,0}$. The Λ CDM model turns out to be an excellent phenomenological model, consistent with our observations. However, certain points remain to be clarified: dark matter and dark energy, which represent the largest fraction of the content of the Universe, are of unknown nature. In the next section, we discuss the observations that currently put the standard model under tension and that could lead to the development of new models.

1.3 Limitations and possible extensions of the standard model

As we have seen in the previous section, the Λ CDM model is consistent with most observations. However, there are a few tensions whose physical or experimental origin has not yet been determined, and which could lead us to new models.

1.3.1 Tensions within the Λ CDM model

The H_0 tension

The tension in the measurement of the current expansion rate of the Universe, the Hubble constant H_0 , is one of the most widely discussed problems in the cosmology literature. It pits two measurements of this parameter against each other. The first measurement is obtained from observations of the CMB, i.e., observations of the young Universe. These measurements of the physics of the Universe at high *redshift* are then propagated using the Λ CDM evolution law, $H(z)$ (Eq. 1.40), to give a constraint on

$H(z=0) = H_0$. One of the most recent measurements is (Planck Collaboration et al. 2020)

$$H_0^{\text{Planck}} = 67.66 \pm 0.42 \text{ km.s}^{-1}.\text{Mpc}^{-1}, \quad (1.88)$$

a measurement with a precision of about $\sim 0.6\%$. The second measurement of H_0 is made using a distance ladder in the local Universe. Measurements of very nearby geometric distances make it possible to calibrate the period-luminosity relation of variable stars called Cepheids. These are used to calibrate the luminosity of more distant SNe Ia whose *redshift* is affected by the expansion of the Universe. This measurement gives us today's value of the Hubble constant (Riess et al. 2022).

$$H_0^{\text{SH0ES}} = 73.04 \pm 1.04 \text{ km.s}^{-1}.\text{Mpc}^{-1}, \quad (1.89)$$

with a precision of $\sim 1.4\%$. Fig. 1.8 shows the evolution of the tension between the two measurements over the years. With the current measurements, the tension is today of order 5σ . An alternative to the SNe Ia measurement uses the tip of the red giant branch (TRGB) to calibrate the distance ladder. The measurement of Freedman et al. 2020, $H_0^{\text{CSP}} = 69.6 \pm 1.7$, is compatible with that from the CMB, while the measurements of Dhawan et al. 2022, $H_0^{\text{ZTF}} = 76.94 \pm 6.4 \text{ km.s}^{-1}.\text{Mpc}^{-1}$, and Scolnic et al. 2023, $H_0^{\text{CATS}} = 73.22 \pm 2.06 \text{ km.s}^{-1}.\text{Mpc}^{-1}$, remain compatible with the Cepheid-calibrated measurements. New independent measurements of comparable precision are beginning to appear. The H0LiCOW collaboration, for example, uses the measurement of time delays between the appearance of quasar images in cases of strong gravitational lensing. One of the most recent measurements is $H_0^{\text{H0LiCOW}} = 73.3 \pm 1.8 \text{ km.s}^{-1}.\text{Mpc}^{-1}$ (Wong et al. 2020). It is in tension with the CMB measurement. This method is expected to yield a measurement of H_0 with a precision of 1% with future data from the *Rubin Observatory*, *Roman*, and *Euclid* (Birrer et al. 2023). Finally, gravitational waves, whose current measurement of H_0 is $H_0^{\text{GW}} = 69^{+16}_{-8} \text{ km.s}^{-1}.\text{Mpc}^{-1}$ (Abbott et al. 2021), are expected to become a more precise probe in the coming years.

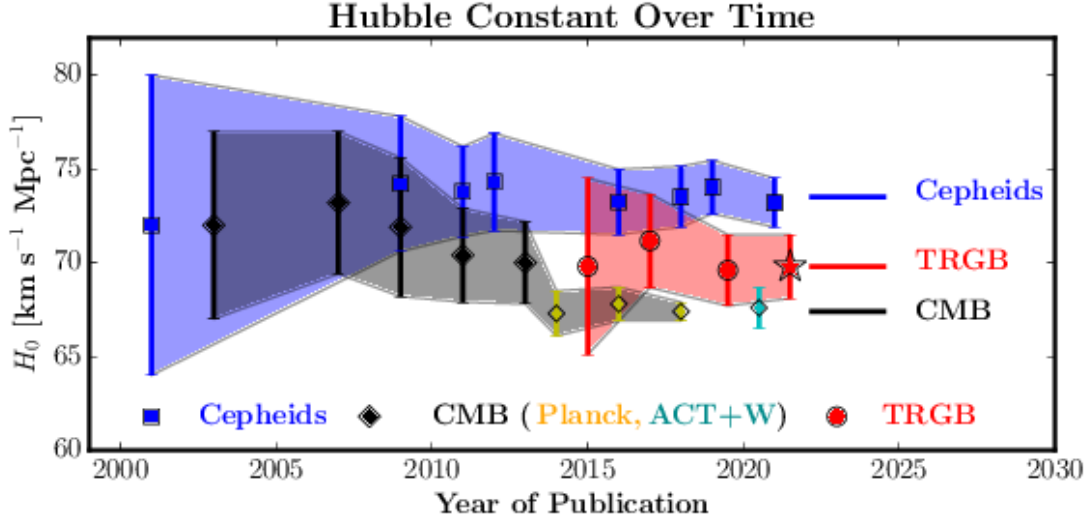


Figure 1.8 – Evolution of the Hubble tension. Credits: arxiv:2106.15656

This tension raises the possibility of a weakness in the Λ CDM model, since it can be understood as the result of an incorrect evolution law for H between the CMB and our epoch. This weakness could open the way to the development of new cosmological models. However, the tension could also be linked to the presence of systematics in the measurements. For SNe Ia, correlations have been mentioned, for example, between luminosity and SNe Ia host properties (Rigault et al. 2015), or a bias in the Cepheid period-luminosity calibration (Efstathiou 2020). For the time being, no consensus has identified systematics that could fully explain the discrepancy between the two values.

The S_8 tension

This tension concerns the observation of large-scale structure. The amplitude of matter clustering in our Universe is quantified by σ_8 , which we will describe in more detail in Chap. 2. This parameter is measured, in particular, through the study of the spatial correlations of galaxies, or through the measurement of weak gravitational lensing effects. The parameter σ_8 is related to the amplitude of the primordial spectrum anisotropies, A_s , and can therefore also be measured with the CMB. This tension is expressed through the parameter

$$S_8 = \sigma_8 \sqrt{\frac{\Omega_m}{0.3}}. \quad (1.90)$$

Note that this tension is also linked to the structure growth-rate parameter $f\sigma_8 \simeq \Omega_m^{0.55} \sigma_8$, which will be discussed in the following chapters. As with H_0 , there is here too a tension between measurements of the young Universe and those of the local

1 Cosmology – 1.3 Limitations and possible extensions of the standard model

Universe. The Planck measurement (Planck Collaboration et al. 2020) gives us

$$S_8^{\text{Planck}} = 0.834 \pm 0.016, \quad (1.91)$$

while the DES measurement, derived from the analysis of the auto-correlations and cross-correlations of weak gravitational lensing and galaxy positions (DES Collaboration et al. 2022), gives

$$S_8^{\text{DES}} = 0.776 \pm 0.017. \quad (1.92)$$

These two measurements are inconsistent at a level of $\sim 2.5\sigma$. This tension does not appear only in the DES data; it is also found in the KiDS-1000 analysis (Heymans et al. 2021), which measures $S_8^{\text{KiDS}} = 0.766^{+0.020}_{-0.014}$ using the same method as DES. A similar tension with Planck is also observed in a measurement using the Lyman- α forests of high-*redshift* quasars from SDSS (Palanque-Delabrouille et al. 2020). Fig. 1.9 gives an overview of various S_8 measurements.

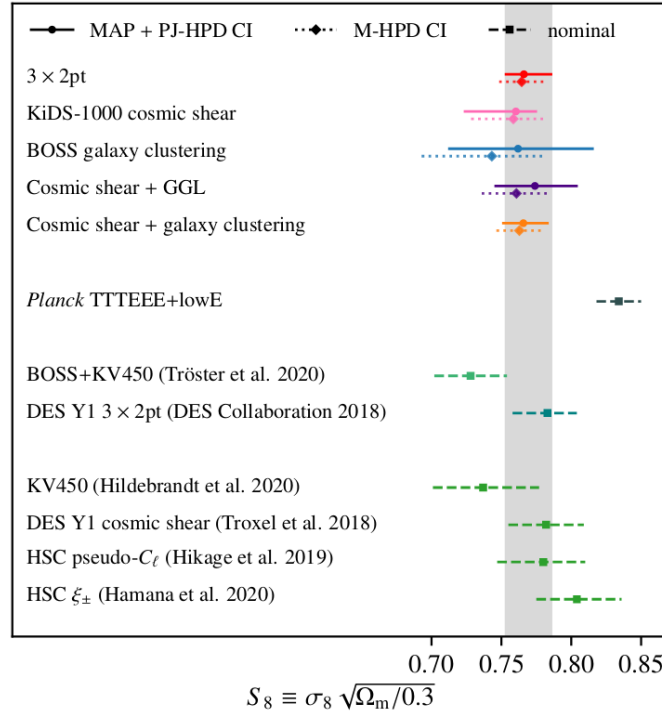


Figure 1.9 – Overview of S_8 measurements. Credits: Heymans et al. 2021

1.3.2 Alternatives to the Λ CDM model

Dark energy models

A first extension of the standard Λ CDM model is to consider that dark energy is not a simple constant. The simplest way to do this is to introduce a new scalar field ϕ ,

described by the Lagrangian

$$\mathcal{L}_{\text{DE}} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi), \quad (1.93)$$

where $V(\phi)$ is a potential energy. The energy-momentum tensor of such a field is given by

$$T_{\mu\nu} = \partial_\mu \phi \frac{\partial \mathcal{L}}{\partial (\partial^\nu \phi)} - g_{\mu\nu} \mathcal{L}. \quad (1.94)$$

We thus obtain the energy density and the pressure

$$\begin{aligned} \rho_\phi &= T_{00} = \frac{1}{2} \dot{\phi}^2 + V(\phi) \\ P_\phi &= T_{ii} = \frac{1}{2} \dot{\phi}^2 - V(\phi). \end{aligned} \quad (1.95)$$

The equation-of-state parameter w is then

$$w = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)}. \quad (1.96)$$

The dynamics of the scalar field is given by Eq. 1.22; we then have for ϕ

$$\ddot{\phi} + 3H\dot{\phi} + \frac{d}{d\phi} V(\phi) = 0. \quad (1.97)$$

Note that if the field evolves slowly, $\dot{\phi} \simeq 0$, we recover the limit of a cosmological constant, $w \simeq -1$. Conversely, if the field varies rapidly, $\ddot{\phi}^2 \gg V(\phi)$, we have $w = 1$; we conclude that for a scalar field $-1 < w < 1$. An equation-of-state parameter $w < -1$ is studied in *phantom dark energy* models and results in a Universe where the scale factor rapidly diverges to infinity; this end of the Universe is called the *Big Rip*. Different choices for the potential $V(\phi)$ lead to different dark energy models. The idea of introducing a cosmological scalar field was proposed before the discovery of the accelerating expansion of the Universe, notably in Wetterich 1988 and Ratra and Peebles 1988. This idea is explored for dark energy under the term *Quintessence* in Caldwell et al. 1998. A class of models of interest today is that of *Early Dark Energy* (EDE), which proposes to resolve the Hubble tension with a dark energy component that intervenes in the early Universe. Constraints in favor of these models have appeared in the analysis of CMB observation data by the ACT collaboration (Hill et al. 2022; La Posta et al. 2022). A review of this class of models is presented in Poulin et al. 2023.

Modified gravity

Another proposed solution to attempt to resolve the tensions of the Λ CDM model and the accelerating expansion of the Universe is to directly modify the laws of gravity. Einstein's general relativity is well tested and constrained up to the scales of the Solar System, but it is possible to construct models that deviate from this theory at larger

scales. One class of such theories is $f(R)$ theories. In $f(R)$ gravity, the Ricci scalar R in the Einstein-Hilbert action (Eq. 1.10) is replaced by a function of it

$$S_{f(R)} = \int d^4x \sqrt{-g} \left[\frac{f(R)}{16\pi G} + \mathcal{L}_m \right]. \quad (1.98)$$

Applying the variational principle then leads to equations different from Einstein's. An extensive review of modified gravity models is presented in Clifton et al. 2012.

A modern approach to modified gravity models is provided by effective field theory (EFT) (Gubitosi et al. 2013; Bloomfield et al. 2013; Piazza and Vernizzi 2013), which, building on Horndeski models (Horndeski 1974), proposes a general parametrization of the general relativity Lagrangian. This parametrization encompasses several models, including $f(R)$, but also scalar-field dark energy theories. The EFT formalism is already implemented in cosmological model solver codes such as HiClass (Zumalacarregui et al. 2017) and EFTCAMB (Hu et al. 2014). It is worth mentioning here that the observation of gravitational waves by the LIGO and VIRGO collaborations (The LIGO Scientific Collaboration and The Virgo Collaboration 2017) has placed significant constraints on the viability of modified gravity models (Amendola et al. 2018b).

These two types of extensions to the Λ CDM model must reproduce the accelerating expansion of the Universe observed in our data. It is fairly difficult to distinguish between modified gravity and dark energy models using a Hubble diagram. However, modified gravity models will more strongly impact the formation and evolution of structures, which is the subject of the next chapter.

2 Velocity field in the inhomogeneous Universe

In Chap. 1 we presented the standard model of Λ CDM cosmology as well as the general aspects of the evolution of the Universe at very large scales. In this framework the Universe was considered homogeneous and isotropic. These hypotheses do not seem to hold at smaller scales: our observations show that matter forms structures under the effect of gravity. Thus, the study of these structures allows us to directly test general relativity and the standard model of cosmology. In this thesis we are particularly interested in the velocity field and in the related growth rate of structures parameter. Modeling the physics at small scales is complex and is still the subject of theoretical developments today. The simplest model for describing this physics is linear perturbation theory. In this chapter we present certain theoretical and observational aspects of cosmology related to the velocity field of the inhomogeneous Universe. In Sec. 2.1 we present linear perturbation theory and the related cosmological parameters. In Sec. 2.2 we detail the observational effects of peculiar velocities as well as some resulting measurements of $f\sigma_8$.

2.1 Linear perturbation theory

Linear perturbation theory provides a first framework for studying the formation of structures and their evolution in the Universe. In this section, we present the theoretical foundations and the cosmological parameters of interest such as the growth factor and the growth rate of structures. This section, too, draws on Peebles 1993, Dodelson and Schmidt 2020 and Jones 2017.

2.1.1 Density contrast and Gaussian random field

Density contrast

As mentioned previously, we are interested here in the inhomogeneities of the Universe. To quantify local deviations from the mean density, we use the scalar field of the "density contrast"

$$\delta(\mathbf{x}, t) = \frac{\rho(\mathbf{x}, t)}{\bar{\rho}(t)} - 1, \quad (2.1)$$

where $\rho(\mathbf{x}, t)$ is the density at each point $\mathbf{x}$ of space at a time t and $\bar{\rho}(t)$ is the density averaged over space at time t .

Fourier space

The study of the density contrast is often carried out in Fourier space. We define the Fourier transform of $\delta(\mathbf{x}, t)$

$$\delta(\mathbf{k}, t) = \int_{-\infty}^{+\infty} d^3\mathbf{x} \delta(\mathbf{x}, t) e^{-i\mathbf{k}\cdot\mathbf{x}} \quad (2.2)$$

$$\delta(\mathbf{x}, t) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3\mathbf{k} \delta(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (2.3)$$

In Fourier space the many differential equations on $\delta(\mathbf{x}, t)$ are expressed more simply, thanks to the relations

$$\nabla \delta(\mathbf{x}, t) \equiv i\mathbf{k} \delta(\mathbf{k}, t) \quad (2.4)$$

$$\nabla^2 \delta(\mathbf{x}, t) \equiv \Delta \delta(\mathbf{x}, t) \equiv -\mathbf{k}^2 \delta(\mathbf{k}, t) \quad (2.5)$$

Gaussian random field, power spectrum and correlation function

Models of initial fluctuations often assume that these fluctuations are described by a Gaussian random field¹. This is a field whose values are spatially correlated, such that if we evaluate these fluctuations $\delta(\mathbf{x})$ at n points $\mathbf{x}_1, \dots, \mathbf{x}_n$ their joint probability is described by a multidimensional normal distribution

$$\mathcal{P}(\delta(\mathbf{x}_1), \dots, \delta(\mathbf{x}_n)) = (2\pi)^{-\frac{n}{2}} |C|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} \boldsymbol{\delta}^T C^{-1} \boldsymbol{\delta} \right], \quad (2.6)$$

where C is the correlation matrix determined by the two-point correlation function $C_{ij} = \xi(\mathbf{x}_i, \mathbf{x}_j) = \xi(\mathbf{x}_i, \mathbf{r}_{ij})$ with $\mathbf{r}_{ij} = \mathbf{x}_j - \mathbf{x}_i$. The homogeneity condition tells us that ξ does not depend on position, so we have $\xi(\mathbf{x}_i, \mathbf{r}_{ij}) = \xi(\mathbf{r}_{ij})$. This correlation function is equal to the covariance of the fluctuations

$$\xi(\mathbf{r}_{ij}) = \langle \delta(\mathbf{x}_i) \delta(\mathbf{x}_j) \rangle. \quad (2.7)$$

Fig. 2.1 presents an illustration of a Gaussian random field.

1. This hypothesis is, at present, confirmed by the measurements of the CMB by the Planck telescope, see Planck Collaboration et al. 2019

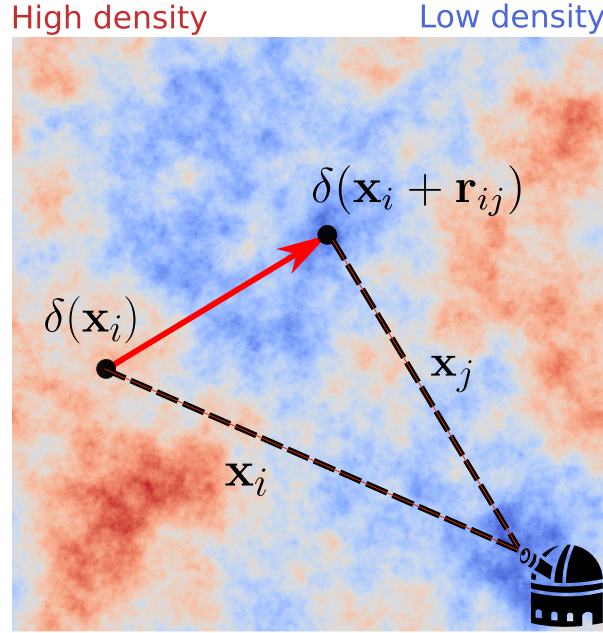


Figure 2.1 – Illustration of a Gaussian random field δ for a power spectrum $P(k) = k^{-3}$. $\mathbf{x}_i$ and $\mathbf{x}_j$ are the vectors between the observer and the two observed points in space, $\mathbf{r}_{ij}$ is the separation vector between $\mathbf{x}_i$ and $\mathbf{x}_j$.

An interesting property of Gaussian random fields is that their correlation function can be entirely encoded in a function in Fourier space called the power spectrum $P(\mathbf{k})$. The power spectrum is defined as the Fourier transform of the two-point correlation function

$$\xi(\mathbf{r}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d^3\mathbf{k} P(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} \quad (2.8)$$

$$P(\mathbf{k}) = \int_{-\infty}^{\infty} d^3\mathbf{r} \xi(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}} \quad (2.9)$$

By definition we can write in Fourier space

$$\langle \delta(\mathbf{k}_i) \delta^*(\mathbf{k}_j) \rangle = \int d^3\mathbf{x}_i \int d^3\mathbf{x}_j \langle \delta(\mathbf{x}_i) \delta(\mathbf{x}_j) \rangle e^{-i\mathbf{k}_i \cdot \mathbf{x}_i} e^{i\mathbf{k}_j \cdot \mathbf{x}_j} \quad (2.10)$$

$$= \int d^3\mathbf{x}_i e^{-i(\mathbf{k}_i - \mathbf{k}_j) \cdot \mathbf{x}_i} \int d^3\mathbf{r}_{ji} \xi(\mathbf{r}_{ji}) e^{-i\mathbf{k}_j \cdot \mathbf{r}_{ji}} \quad (2.11)$$

$$= (2\pi)^3 \delta_D(\mathbf{k}_i - \mathbf{k}_j) P(\mathbf{k}_i), \quad (2.12)$$

where δ_D is the Dirac distribution. We note that in Fourier space, the covariance of the $\mathbf{k}$ modes is zero and their variance is $\sigma_{\mathbf{k}}^2 = (2\pi)^3 P(\mathbf{k})$. The isotropy condition leads to a power spectrum that depends only on the norm of the modes $P(\mathbf{k}) \equiv P(k)$ with

$k = |\mathbf{k}|$. We then have

$$\xi(r) = \frac{1}{2\pi^2} \int_0^{+\infty} dk k^2 P(k) \frac{\sin kr}{kr}. \quad (2.13)$$

Variance of the matter field

In linear theory we assume that the fluctuations of the matter field δ_m can still be described by a Gaussian random field. It is conventional to look at these fluctuations averaged within a certain window described by a function $W(\mathbf{x})$. Indeed, by averaging observations over a certain volume, we remove the small-scale nonlinearities. We can, for example, use a spherical window function of radius R

$$W_R(\mathbf{x}) = \begin{cases} 1 & \text{if } |\mathbf{x}| < R \\ 0 & \text{otherwise} \end{cases} \quad (2.14)$$

The Fourier transform of this window function is

$$W_R(k) = \frac{3j_1(kR)}{kR}, \quad (2.15)$$

where j_1 is the spherical Bessel function such that

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}. \quad (2.16)$$

The variance of the matter field in a sphere of radius R is then given by

$$\langle \delta^2(\mathbf{x}) \rangle_R = \sigma_R^2 = \frac{1}{2\pi^2} \int \frac{dk}{k} P_{\delta\delta}(k) \left(\frac{3j_1(kR)}{kR} \right)^2. \quad (2.17)$$

Historically, the variance is used in spheres of radius $R = 8 \text{ Mpc} \cdot h^{-1}$, quantified by the parameter σ_8 . Although sometimes criticized (Sanchez 2020), σ_8 remains very widely used in cosmology. This parameter is proportional to the amplitude of the power spectrum, and we can define a normalized power spectrum

$$\tilde{P}_{\delta\delta}(k) = \frac{P_{\delta\delta}(k)}{\sigma_8^2}. \quad (2.18)$$

2.1.2 Perturbed Einstein equations

Perturbation of the FLRW metric

The hypotheses of isotropy and homogeneity no longer hold, so the FLRW metric is no longer valid here. We must allow for perturbations of it. In general the perturbed FLRW metric is written

$$ds^2 = a^2(\eta) \left(-[1 + h_{00}] d\eta^2 + h_{0i} dx^i d\eta + [\gamma_{ij} + h_{ij}] dx^i dx^j \right), \quad (2.19)$$

2 Velocity field in the inhomogeneous Universe – 2.1 Linear perturbation theory

where η is the conformal time such that $d\eta = a(t)dt$, γ_{ij} is the spatial part of the FLRW metric from Eq. 1.15 and $h_{\mu\nu}$ represents the perturbations. In general the components of h can be written

$$h_{00} = 2A \quad (2.20)$$

$$h_{0i} = B_i \quad (2.21)$$

$$h_{ij} = -2C\gamma_{ij} + 2D_{ij} \quad (2.22)$$

where A is a scalar potential, B_i a vector, C_i a scalar perturbation of the spatial curvature and D_{ij} a traceless spatial distortion ($\gamma^{ij}D_{ij} = 0$). This perturbed metric can be decomposed into three types of perturbations: scalar, vector and tensor. The vector B_i is decomposed into two terms

$$B_i = B_{\parallel,i} + B_{\perp,i}, \quad (2.23)$$

which correspond respectively to the curl-free component $\nabla \times B_{\parallel} = 0$ and the divergence-free component $\nabla \cdot B_{\perp} = \gamma^{ij}\nabla_i B_{\perp,j} = 0$. The curl-free part can be re-expressed in terms of a scalar

$$B_{\parallel,i} = \nabla_i B. \quad (2.24)$$

A similar decomposition is carried out with the tensor D_{ij}

$$D_{ij} = \left(\partial_i \partial_j - \frac{1}{3} \gamma_{ij} \partial_p \partial^p \right) D + (\nabla_i D_{\perp,j} + \nabla_j D_{\perp,i}) + \bar{D}_{ij}, \quad (2.25)$$

where D is a scalar, $D_{\perp}$ a divergence-free vector and $\bar{D}$ the traceless and divergence-free component of D . This decomposition, abbreviated SVT (Scalar-Vector-Tensor), is widely used in perturbation theory; it is described in more detail in Bardeen 1980; Hu and Eisenstein 1999; Kodama and Sasaki 1984.

The scalar perturbations A, B, C and D are directly related to variations in energy density, the vector perturbations $B_{\perp}$ and $D_{\perp}$ generally decay rapidly in the early Universe, and the tensor perturbations $\bar{D}$ are the origin of gravitational waves. The *decomposition theorem*, which follows from the isotropy of the FLRW metric, implies that at first order, the fluctuations evolve independently. Here we focus solely on the study of scalar perturbations, since these dominate the formation of structures.

In General Relativity, the physical equations are independent of the choice of coordinates, called the choice of gauge, which allows the number of degrees of freedom of the equations to be reduced. In linear perturbation theory, we consider coordinate transformations small enough to remain within the linear regime. It can be shown that, for scalar perturbations, we can go from the four scalar functions A, B, C and D to two gauge-invariant functions. The choice of these two functions is not unique; a widely used choice is that of the two Bardeen potentials in the *conformal Newtonian gauge* ($B = D = 0, A = \psi, C = \phi$). This allows the metric to be rewritten

$$ds^2 = a^2(\eta) \left(-(1 + 2\psi) d\eta^2 + (1 - 2\phi) \gamma_{ij} dx^i dx^j \right), \quad (2.26)$$

where the two scalar potentials $\psi(\mathbf{x}, t)$ and $\phi(\mathbf{x}, t)$ can respectively be interpreted as the Newtonian gravitational potential and a perturbation of the scale factor.

Perturbation of the energy-momentum tensor

In the same gauge, the perturbed energy-momentum tensor is written

$$T^0_0 = -\rho(t) (1 + \delta) \quad (2.27)$$

$$T^0_i = (\rho + P) v_i \quad (2.28)$$

$$T^i_j = (P + \delta P) \delta^i_j + P \Pi^i_j \quad (2.29)$$

where v_i is the velocity vector $v_i = \frac{dx^i}{d\eta}$ and Π^i_j is the anisotropic stress tensor. In the same way as before, the vector v_i and the tensor Π_{ij} can be decomposed into SVT terms.

$$v_i = \partial_i v + v_{\perp,i} \quad (2.30)$$

$$\Pi_{ij} = \left(\partial_i \partial_j - \frac{1}{3} \gamma_{ij} \partial_p \partial^p \right) \Pi + (\nabla_i \Pi_{\perp,j} + \nabla_j \Pi_{\perp,i}) + \bar{\Pi}_{ij}. \quad (2.31)$$

These various perturbation terms can be related to the different species contained in the Universe by solving the corresponding Boltzmann equations (Eq. 1.28).

Einstein equations for the perturbations

The Einstein equations, which in the homogeneous regime yield the Friedmann equations (Eqs. 1.38 and 1.39), will now provide additional equations for the perturbations. Here, for simplicity, we choose to work in flat space $\gamma_{ij} = \delta_{ij}$. Moreover, we will move to Fourier space. It can be shown that the perturbations from the SVT decomposition can be expressed in terms of the eigenvectors of the Laplacian operator $\Delta Q = -k^2 Q$ (Bardeen 1980; Kodama and Sasaki 1984). For example, in flat space $Q = e^{i\mathbf{k}\cdot\mathbf{x}}$. The scalar decompositions of the perturbations are then expressed as

$$s(x) \equiv s(k) Q \quad \text{for scalars} \quad (2.32)$$

$$\partial_i v(x) \equiv k^{-1} v(k) \partial_i Q \quad \text{for vectors} \quad (2.33)$$

$$\left(\partial_i \partial_j - \frac{1}{3} \gamma_{ij} \partial_p \partial^p \right) t(x) \equiv k^{-2} t(k) \left(\partial_i \partial_j + \frac{1}{3} \gamma_{ij} k^2 \right) Q \quad \text{for tensors} \quad (2.34)$$

$$(2.35)$$

With this formalism, the $\delta G^i_j = 8\pi G \delta T^i_j$ component of the Einstein equation for scalar perturbations is

$$-k^2 (\psi(k) - \phi(k)) = 8\pi G a^2 P \Pi(k), \quad (2.36)$$

In the case of a perfect fluid, the anisotropic stress is negligible $\Pi \simeq 0$ and we then have $\psi = \phi$. The term $\delta G^0_0 = 8\pi G \delta T^0_0$ gives us

$$-k^2 \phi(k) = 4\pi G a^2 \left(\rho \delta(k) + \mathcal{H}(\rho + P) \frac{v(k)}{k} \right). \quad (2.37)$$

At scales below the Hubble radius, such that $k\mathcal{H}^{-1} \gg 1$, this equation tends toward the Poisson equation

$$-k^2 \phi = 4\pi G a^2 \rho \delta \quad (2.38)$$

$$\Leftrightarrow \Delta \phi = 4\pi G a^2 \rho \delta. \quad (2.39)$$

2.1.3 Growth factor of structures

Evolution of the primordial perturbations

We distinguish different periods in the evolution of the fluctuations. In the early Universe, after the big bang, the modes are outside the Hubble radius and do not evolve. The expansion then gradually slows down and the modes progressively enter the Hubble radius and begin to evolve and form matter structures. During the matter-domination phase, all modes evolve identically. In the present-day Universe, dark energy and the acceleration of the expansion cause the progressive decay of the different modes. We relate the present-day potential $\phi(k, a)$ to the primordial fluctuations $\mathcal{R}$ by

$$\phi(k, a) = \frac{3}{5} \mathcal{R}(k) T(k) \frac{D_+(a)}{D_+(a_{\text{matter}})}, \quad (2.40)$$

where $T(k)$ is called the transfer function, $D_+(a)$ the growth factor and a_{matter} the scale factor chosen at a time when matter dominates. The transfer function is defined by

$$T(k) = \frac{\phi(k, a_{\text{matter}})}{\phi(k \rightarrow 0, a_{\text{matter}})}. \quad (2.41)$$

Using the Poisson equation for scales below the Hubble radius we obtain the relation for the matter fluctuation

$$\delta_m(k, a) = \frac{2}{5} \frac{k^2}{\Omega_{m,0} H_0^2} \mathcal{R}(k) T(k) D_+(a). \quad (2.42)$$

Using Eq. 1.77 we obtain the power spectrum

$$P_{\delta\delta}(k) = \frac{8\pi^2}{25} \frac{\mathcal{A}_s}{\Omega_{m,0}^2 H_0^4} T^2(k) D_+^2(a) \frac{k^{n_s}}{k_0^{n_s-1}} \quad (2.43)$$

Evolution of the growth factor

In general, the equations of motion are obtained from the conservation of the energy-momentum tensor $\nabla_\mu T^\mu_\nu = 0$. In the case of the non-relativistic matter fluctuations δ_m , and at scales below the Hubble radius, we can adopt a Newtonian approach. We consider comoving coordinates $\mathbf{x}$ such that for an object at a physical distance $\mathbf{r}$ we have

$$\mathbf{x} = \frac{\mathbf{r}}{a}. \quad (2.44)$$

We then relate the gradient with respect to $\mathbf{r}$ and that with respect to $\mathbf{x}$

$$\nabla_{\mathbf{x}} = a \nabla_{\mathbf{r}}. \quad (2.45)$$

The peculiar velocity $\mathbf{v}$ is related to the physical velocity $\mathbf{V}$ by

$$\mathbf{V} = \dot{a}\mathbf{x} + \mathbf{v}(\mathbf{x}, t), \quad (2.46)$$

where $\dot{a}\mathbf{x} = H\mathbf{r}$ is the velocity due to the expansion. The evolution equations are

$$\partial_t \delta_m + \frac{1}{a} \nabla_{\mathbf{x}} [(1 + \delta)\mathbf{v}] = 0 \quad \text{Continuity equation} \quad (2.47)$$

$$\partial_t \mathbf{v} + H\mathbf{v} + \frac{1}{a} (\mathbf{v} \cdot \nabla_{\mathbf{x}}) \mathbf{v} = -\frac{1}{a} \nabla_{\mathbf{x}} \phi - \frac{c_s^2}{a} \nabla_{\mathbf{x}} \delta_m \quad \text{Euler equation} \quad (2.48)$$

where $c_s^2 = \frac{\partial P}{\partial \rho}$ is the sound speed. Since we are considering small variations, we can linearize these two equations by neglecting all terms of order higher than one: $\delta^2, v^2, \delta v$. We then obtain the equations

$$\partial_t \delta_m + \frac{1}{a} \nabla_{\mathbf{x}} \cdot \mathbf{v} = 0 \quad (2.49)$$

$$\partial_t \mathbf{v} + H\mathbf{v} = -\frac{1}{a} \nabla_{\mathbf{x}} \phi - \frac{c_s^2}{a} \nabla_{\mathbf{x}} \delta_m. \quad (2.50)$$

If we combine Eqs. 2.49 and 2.50 with the Poisson equation (Eq. 2.39) we obtain

$$\partial_t^2 \delta_m + 2H\partial_t \delta_m = 4\pi G\rho_m \delta_m + \frac{c_s^2}{a^2} \Delta_{\mathbf{x}} \delta_m. \quad (2.51)$$

In Fourier space

$$\partial_t^2 \delta_m + 2H\partial_t \delta_m + 4\pi\rho_m \left(\frac{k^2 c_s^2}{4\pi\rho a^2} - 1 \right) \delta_m = 0. \quad (2.52)$$

We call the quantity $\lambda_J = 2\pi a k_J^{-1} = \sqrt{\pi(G\rho)^{-1}} c_s$ the Jeans length, which characterizes the scale at which pressure forces are strong enough to oppose gravitational collapse. If $k < k_J$, we are in the case of gravitational collapse and the solution for δ is growing. In the second case $k > k_J$ the pressure forces are stronger than gravity, and the solutions

2 Velocity field in the inhomogeneous Universe – 2.1 Linear perturbation theory

for δ are oscillating solutions. At scales larger than the Jeans scale $k \gg k_J$, or for pressureless fluids such as dark matter, we can neglect the pressure term and obtain the equation

$$\partial_t^2 \delta_m + 2H\partial_t \delta_m = 4\pi G\rho_m \delta_m. \quad (2.53)$$

Furthermore, if we replace ρ_m with the dimensionless parameter Ω_m , we obtain

$$\partial_t^2 \delta_m + 2H\partial_t \delta_m - \frac{3}{2}H_0^2 \Omega_{m,0} a^{-3} \delta_m = 0. \quad (2.54)$$

The solutions of this equation are of the form

$$\delta_m(\mathbf{x}, t) = \hat{\delta}_m(\mathbf{x}) (D_+(t) + D_-(t)), \quad (2.55)$$

where $D_+(t)$ is a growing solution and $D_-(t)$ a decaying solution. By applying Eq. 2.54 to $H(t)$ one can show that, in the case of the Λ CDM model with zero curvature, the decaying solution is $D_-(t) \propto H(t)$. When $D_+(t)$ dominates we have $\delta_m(\mathbf{x}, t) = \hat{\delta}_m(\mathbf{x})D_+(t)$ and Eq. 2.54 becomes

$$\partial_t^2 D_+ + 2H\partial_t D_+ - \frac{3}{2}H_0^2 \Omega_{m,0} a^{-3} D_+ = 0. \quad (2.56)$$

In the matter-dominated regime it can be shown that the solution to Eq. 2.56 is $D_+(a) = a$. In the case of the present-day Universe, with matter and a cosmological constant, the solution of this equation is

$$D_+(a) = \frac{5}{2}\Omega_{m,0}E(a) \int_0^a \frac{da'}{[a'E(a')]^3}, \quad (2.57)$$

where $E(a)$ is the function defined by $E(a) = H(a)H_0^{-1}$ and the integration constant is fixed such that $\lim_{a \rightarrow \infty} D_+(a) = a$.

From the point of view of the power spectrum we have

$$\langle \delta(\mathbf{k}, a) \delta(\mathbf{k}', a) \rangle = \frac{D_+^2(a)}{D_+^2(a_0)} \langle \delta(\mathbf{k}, a_0) \delta(\mathbf{k}', a_0) \rangle \quad (2.58)$$

$$= \frac{D_+^2(a)}{D_+^2(a_0)} (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P_{\delta\delta}(k; a_0) \quad (2.59)$$

$$= \frac{D_+^2(a)}{D_+^2(a_0)} \sigma_{8,0}^2 (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') \tilde{P}_{\delta\delta}(k; a_0). \quad (2.60)$$

From this we deduce the general expression for σ_8

$$\sigma_8(a) = \sigma_{8,0} \frac{D_+(a)}{D_+(a_0)}. \quad (2.61)$$

2.1.4 Velocities and the growth rate of structures

Velocity field

If we return to our linearized continuity equation (Eq. 2.49) we have

$$\nabla \cdot \mathbf{v} = -a \partial_t \delta_m \quad (2.62)$$

$$= -\dot{a} \frac{a}{D_+(a)} \frac{dD_+(a)}{da} \delta_m \quad (2.63)$$

$$= -aHf \delta_m \quad (2.64)$$

with f the growth rate of structures

$$f = \frac{d \ln D_+}{d \ln a}. \quad (2.65)$$

The solution of this equation is of the form

$$\mathbf{v}(\mathbf{x}) = \frac{aHf}{4\pi} \int \frac{\mathbf{x}' - \mathbf{x}}{|\mathbf{x}' - \mathbf{x}|^3} \delta_m(\mathbf{x}') d^3\mathbf{x}'. \quad (2.66)$$

We often define the variable θ such that

$$\theta(\mathbf{x}) = -\frac{1}{aHf} \nabla \cdot \mathbf{v}(\mathbf{x}), \quad (2.67)$$

in the linear regime we have $\theta = \delta$. Since, as discussed in Sec. 2.1.2, the vector perturbations decay rapidly, we can posit that the velocity derives from a potential $v(\mathbf{x})$ such that

$$\mathbf{v}(\mathbf{x}) = \nabla v(\mathbf{x}). \quad (2.68)$$

This allows us to write, in Fourier space,

$$v(\mathbf{k}) = -\frac{aHf}{k^2} \theta(\mathbf{k}) \quad (2.69)$$

if we take the gradient of this last equation

$$\mathbf{v}(\mathbf{k}) = -iaHf \frac{\mathbf{k}}{k^2} \theta(\mathbf{k}). \quad (2.70)$$

Growth rate of structures

We can return to the growth rate of structures that appears in Eq. 2.65. For the matter-dominated period, we obtain that, since $D_+(a) = a$, then $f = 1$. For a Universe with matter and dark energy, the derivative of Eq. 2.57 gives

$$f(a) = \frac{\Omega_{m,0}}{a^3 E^2(a)} \left[\frac{5a}{2D_+(a)} - \frac{3}{2} \right]. \quad (2.71)$$

Fig. 2.2 shows the evolution of f , σ_8 and $f\sigma_8$ as a function of *redshift* z , within the Λ CDM model.

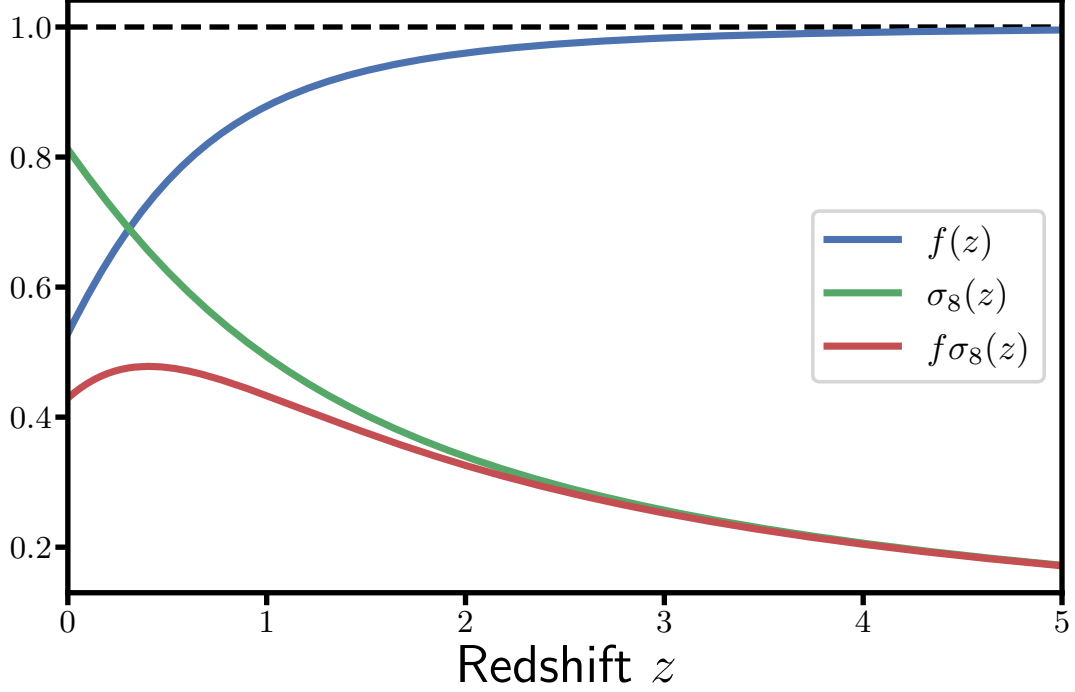


Figure 2.2 – Evolution of the parameters σ_8 , f and $f\sigma_8$ as a function of redshift z .

We can re-express Eq. 2.56 in terms of f

$$\frac{d}{d \ln a} f + f^2 + \left(\frac{\dot{H}}{H} + 2 \right) f = \frac{3}{2} \Omega_M(a), \quad (2.72)$$

where Ω_M is defined by $\Omega_M(a) = \Omega_m(a)/H^2(a) = \Omega_{m,0} a^{-3} E^{-2}(a)$. In the Λ CDM model dominated by matter and the cosmological constant we have

$$\frac{d\Omega_M}{d \ln a} = -3\Omega_M(1 - \Omega_M). \quad (2.73)$$

We can rewrite Eq. 2.72 as

$$-3\Omega_M(1 - \Omega_M) \frac{d}{d\Omega_M} f + f^2 + \left(2 - \frac{3}{2}\Omega_M \right) f = \frac{3}{2}\Omega_M \quad (2.74)$$

A simple parametrization of the growth rate of structures is then proposed as

$$f \simeq \Omega_M^\gamma, \quad (2.75)$$

where γ is called the growth index. By inserting Eq. 2.75 into Eq. 2.74 and keeping only

first-order terms in $1 - \Omega_M$ we obtain (Wang and Steinhardt 1998)

$$\gamma \simeq \frac{6}{11} + \frac{15}{1331}(1 - \Omega_M). \quad (2.76)$$

In the limit of a matter-dominated universe, $\Omega_M = 1$ and we have $\gamma \simeq \frac{6}{11} \simeq 0.55$. This value, or a value close to it, $\gamma = 0.6$, is found in numerous studies (e.g. Peebles 1993); it represents the value expected for the standard GR + Λ CDM model. While in general relativity the growth rate of structures f is independent of scale k , this is not always the case for modified gravity theories, such as $f(R)$ theories for example (Narikawa and Yamamoto 2010).

2.2 Observational consequences

2.2.1 Doppler effect and relativistic aberration

Relativistic Doppler effect

Photons emitted by an astrophysical object will not be observed in the same way in every reference frame. Indeed, in special relativity the velocity between the source's reference frame and the observer's gives rise to the relativistic Doppler effect. To demonstrate this, we can imagine an observer at rest with respect to the CMB and close to an astrophysical source S that emits photons isotropically in its own rest frame (Fig. 2.3). To simplify the calculations we will use the natural unit system ($c = \hbar = 1$). In this unit system, the astrophysical object has a velocity $\mathbf{v}$, and we set up our frame such that $\mathbf{v} = v\mathbf{e}_x$ and the line of sight $\mathbf{n}$ lies in the plane (O, x, y) . If we denote by θ the angle between $\mathbf{v}$ and $\mathbf{n}$ in the source's reference frame, we have

$$\mathbf{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}. \quad (2.77)$$

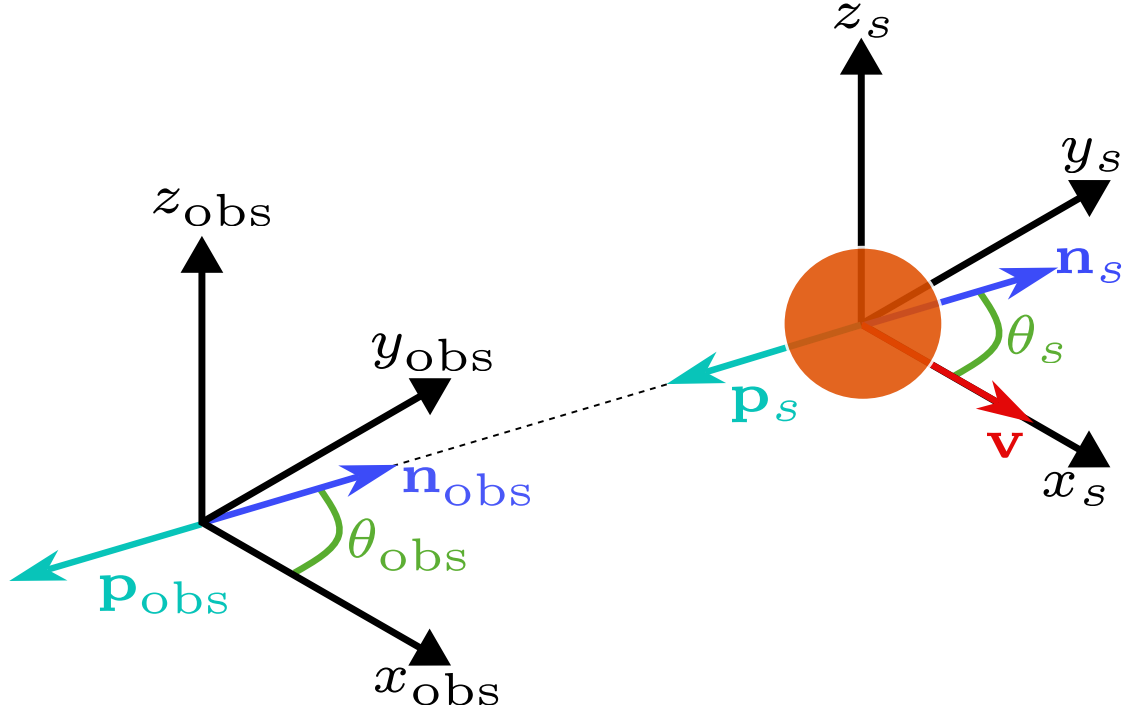


Figure 2.3 – Diagram of the reference frame of the source S moving at velocity v , relative to that of the comoving observer. The vectors $\mathbf{p}_{s/\text{obs}}$ represent the spatial part of the 4-vector momentum of the photons in the reference frames of the source and the observer, the vector $\mathbf{n}_{\text{obs}}$ is the observer's line of sight, and the vector $\mathbf{n}_s$ is an analogous vector in the source's reference frame.

Locally, we can describe spacetime by a Minkowski metric $\eta_{\mu\nu} = (-dt, dx^0, dx^1, dx^2)$. The propagation of a photon of frequency ν emitted toward the observer is then represented by its 4-momentum vector

$$p_s^\mu = \begin{pmatrix} \omega_s \\ -\omega_s \mathbf{n}_s \end{pmatrix} = \begin{pmatrix} \omega_s \\ -\omega_s \cos \theta_s \\ -\omega_s \sin \theta_s \\ 0 \end{pmatrix}, \quad (2.78)$$

where $\omega_s = 2\pi\nu$.

To see how this photon is perceived by the observer we must apply the inverse Lorentz boost corresponding to velocity v

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & v\gamma & 0 & 0 \\ v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (2.79)$$

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where γ is the Lorentz factor

$$\gamma \equiv \gamma(v) = \frac{1}{\sqrt{1-v^2}} \quad (2.80)$$

We then obtain the photon's momentum vector in the observer's reference frame

$$p_{\text{obs}}^\mu = \Lambda^\mu{}_\nu p_s^\nu = \begin{pmatrix} \omega_s \gamma (1 - v \cos \theta_s) \\ \omega_s \gamma (v - \cos \theta_s) \\ -\omega_s \sin \theta_s \\ 0 \end{pmatrix}. \quad (2.81)$$

By identification we can see that the observed pulsation ω_{obs} is written

$$\omega_{\text{obs}} = \omega_s \gamma (1 - v \cos \theta_s) \quad (2.82)$$

and the angle at which the photon is observed is

$$\cos \theta_{\text{obs}} = -\frac{p_{\text{obs}}^1}{p_{\text{obs}}^0} = \frac{\cos \theta_s - v}{1 - v \cos \theta_s} \quad (2.83)$$

We can invert Eq. 2.83 to obtain $\cos \theta_s$

$$\cos \theta_s = \frac{\cos \theta_{\text{obs}} + v}{1 + v \cos \theta_{\text{obs}}}. \quad (2.84)$$

By inserting Eq. 2.84 into Eq. 2.82 we have

$$\omega_{\text{obs}} = \omega_s \gamma \left(1 - v \frac{\cos \theta_{\text{obs}} + v}{1 + v \cos \theta_{\text{obs}}} \right) \quad (2.85)$$

$$= \omega_s \gamma \frac{1 - v^2}{1 + v \cos \theta_{\text{obs}}} \quad (2.86)$$

$$= \omega_s \frac{\sqrt{1 - v^2}}{1 + \mathbf{v} \cdot \mathbf{n}_{\text{obs}}}. \quad (2.87)$$

We can finally obtain the ratio between λ_{obs} and λ_s

$$\frac{\lambda_{\text{obs}}}{\lambda_s} = \frac{\omega_s}{\omega_{\text{obs}}} = \frac{1 + \mathbf{v} \cdot \mathbf{n}_{\text{obs}}}{\sqrt{1 - v^2}}. \quad (2.88)$$

We then have an expression for the *redshift* due to peculiar velocities

$$1 + z_p = \frac{1 + \mathbf{v} \cdot \mathbf{n}_{\text{obs}}}{\sqrt{1 - v^2}}. \quad (2.89)$$

A first-order approximation in v of Eq. 2.89 gives us $(c)z_p \simeq \mathbf{v} \cdot \mathbf{n}_{\text{obs}} \simeq v_p$, where the notation v_p will be used throughout the rest of this manuscript to refer to the line-of-sight velocity. The term *redshift* is here something of a misnomer. Indeed, the

projection of the velocity onto the line of sight can be positive or negative, so we can have a shift toward the red or toward the blue.

The light received from a source comes from a small solid angle Ω_s . Since the angle ϕ between the plane $(0, x, y)$ and the z axis is not modified by the Lorentz boost, Ω_s depends only on $d \cos \theta_s$. We can find the relation between $d \cos \theta_s$ and $d \cos \theta_{\text{obs}}$ by returning to the result of Eq. 2.83

$$d \cos \theta_{\text{obs}} = \frac{d \cos \theta_s}{1 - v \cos \theta_s} + v \frac{\cos \theta_s - v}{(1 - v \cos \theta_s)^2} d \cos \theta_s \quad (2.90)$$

$$= \frac{1 - v^2}{(1 - v \cos \theta_s)^2} d \cos \theta_s. \quad (2.91)$$

Using Eq. 2.84 in Eq. 2.91 we obtain

$$d \cos \theta_{\text{obs}} = \frac{(1 - v^2)(1 + v \cos \theta_{\text{obs}})^2}{(1 - v^2)^2} d \cos \theta_s \quad (2.92)$$

$$= \frac{(1 + v \cos \theta_{\text{obs}})^2}{1 - v^2} d \cos \theta_s \quad (2.93)$$

$$= \frac{(1 + \mathbf{v} \cdot \mathbf{n})^2}{1 - v^2} d \cos \theta_s \quad (2.94)$$

$$= (1 + z_p)^2 d \cos \theta_s. \quad (2.95)$$

We then have the relation between the solid angles

$$\frac{\Omega_{\text{obs}}}{\Omega_s} = (1 + z_p)^2. \quad (2.96)$$

Composition of *redshifts*

We can use the previous results to obtain the expression for the *redshift* measured by an observer far from the source. If we first take an observer 1, at rest with respect to the CMB reference frame and close enough to the source S to not be sensitive to the expansion of the Universe. Since the source has a velocity relative to observer 1, the latter measures a Doppler effect (Eq. 2.88) such that the photons emitted by the source appear to them with a wavelength

$$\lambda_1 = (1 + z_p) \lambda_s, \quad (2.97)$$

where λ_s is the wavelength of the photons at the time of their emission and z_p the *redshift* due to the peculiar velocity of the source relative to observer 1. We now take a second observer, denoted 2, at rest with respect to the CMB, but this time located at a sufficient distance from observer 1 to feel the effects of the cosmological expansion. Observer 2 measures the same photons as observer 1 with a wavelength

$$\lambda_2 = (1 + z_{\text{cos}}) \lambda_1 = (1 + z_{\text{cos}})(1 + z_p) \lambda_s, \quad (2.98)$$

where we denote by z_{cos} the *redshift* due to the expansion of the Universe. Finally, we take a last observer close to observer 2, but with a velocity $\mathbf{v}_{\text{obs}}$ relative to the CMB reference frame. Reasoning analogous to the derivation of the Doppler effect between observer 1 and the source gives

$$1 + z_{p,\text{obs}} = \frac{1 - \mathbf{v}_{\text{obs}} \cdot \mathbf{n}}{\sqrt{1 - v_{\text{obs}}^2}}. \quad (2.99)$$

We then have the wavelength measured by the last observer

$$\lambda_{\text{obs}} = (1 + z_{p,\text{obs}})(1 + z_{\text{cos}})(1 + z_p)\lambda_s, \quad (2.100)$$

from which we derive the expression for the observed *redshift*

$$1 + z_{\text{obs}} = (1 + z_{p,\text{obs}})(1 + z_{\text{cos}})(1 + z_p). \quad (2.101)$$

Relativistic aberration and luminosity distance

We saw in Eq. 2.82 that the solid angle of emission of the photons is modified by the velocity of the source. Using the redshift composition relation (Eq. 2.101), the luminosity distance from Eq. 1.73 can be rewritten

$$d_{L,\text{obs}} = (1 + z_{\text{obs}}) \sqrt{\frac{A_{\text{obs}}}{\Omega_s}} \quad (2.102)$$

$$= (1 + z_{\text{obs}}) \sqrt{\frac{\Omega_1}{\Omega_s} \frac{A_{\text{obs}}}{\Omega_1}}, \quad (2.103)$$

where Ω_1 corresponds to the solid angle seen by an observer at rest with respect to the CMB, close to the source. The ratio $\frac{\Omega_1}{\Omega_s}$ corresponds to a situation identical to that of Eq. 2.82 and $\frac{A_{\text{obs}}}{\Omega_1}$ corresponds to the situation used to derive Eq. 1.73. We thus have

$$d_{L,\text{obs}} = (1 + z_{\text{obs}})(1 + z_p)S_k(\chi) = (1 + z_{p,\text{obs}})(1 + z_p)^2 d_{L,\text{cos}}. \quad (2.104)$$

We note that we have not taken into account any correction to A_{obs} due to the observer's velocity. Indeed, these corrections are negligible at first order (Hui and Greene 2006).

2.2.2 Redshift-space distortion

Redshift space

The relativistic Doppler effect due to velocities directly affects our observations. Its impacts on the distribution of galaxies are in particular derived in Kaiser 1987. During *redshift* surveys, the measured *redshift* of each galaxy is modified by that galaxy's

velocity. The distance to a galaxy estimated using this *redshift* is then, at first order,

$$r_{\text{obs}} = r_{\text{cos}} + \frac{v_{\parallel}(\mathbf{x}_{\text{cos}})}{aH}. \quad (2.105)$$

where $v_{\parallel}(\mathbf{x}_{\text{cos}}) = \mathbf{v}(\mathbf{x}_{\text{cos}}) \cdot \hat{\mathbf{x}}_{\text{cos}}$ is the velocity projected onto the line of sight. This is equivalent to having a position vector

$$\mathbf{x}_{\text{obs}} = \mathbf{x}_{\text{cos}} + \frac{v_{\parallel}(\mathbf{x}_{\text{cos}})}{aH} \hat{\mathbf{x}}_{\text{cos}}. \quad (2.106)$$

We refer to this space of deformed distances as *redshift space*. The effect of velocities on the *redshift* is represented in Fig. 2.4. We distinguish two regimes. The first, on the left, is that of large scales, where a linear description can be used. Galaxies in front of and behind an overdensity fall toward it. The *redshifts* are modified by the velocity of this motion and the distances then appear "squashed" along the line of sight; this is called the Kaiser effect. The second regime, shown on the right, is the nonlinear one that takes place at small scales. Near the overdensity, velocities become large. The large velocity dispersion then creates an elongation effect in *redshift space* known as the "finger of God" effect. These effects are generally referred to as redshift-space distortions (RSD).

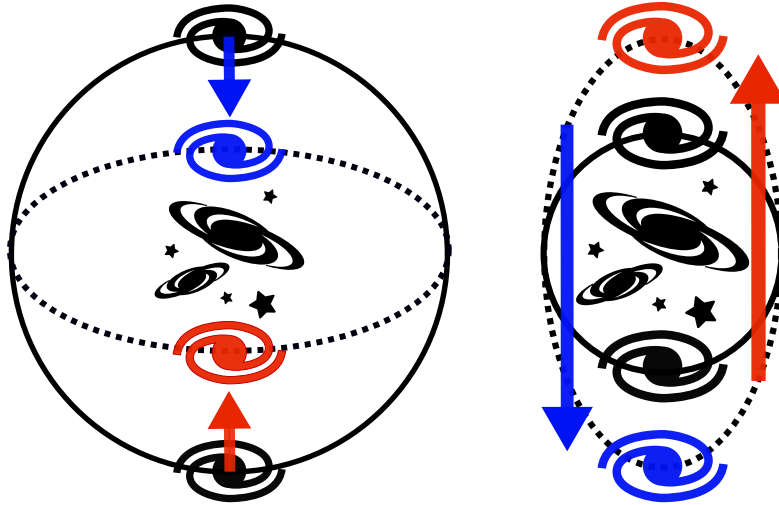


Figure 2.4 – Illustration of redshift-space distortions. The solid circle represents real space, the dashed one redshift space. The arrows represent the line-of-sight velocity. In blue, the position of a galaxy with velocity directed toward us, in red that of a galaxy with velocity directed in the opposite direction. *Left:* at large scales we observe a squashing of redshift space, this is the Kaiser effect. *Right:* at smaller scales, the velocities stretch redshift space along the line of sight; this is called the "finger of God" effect.

Fig. 2.5 shows an estimate of the correlation function of galaxy pairs from the 2dFGRS survey (Peacock et al. 2001). This is a histogram of galaxy pairs as a function

of their radial and line-of-sight distance. Both effects just described appear here, at large and small scales.

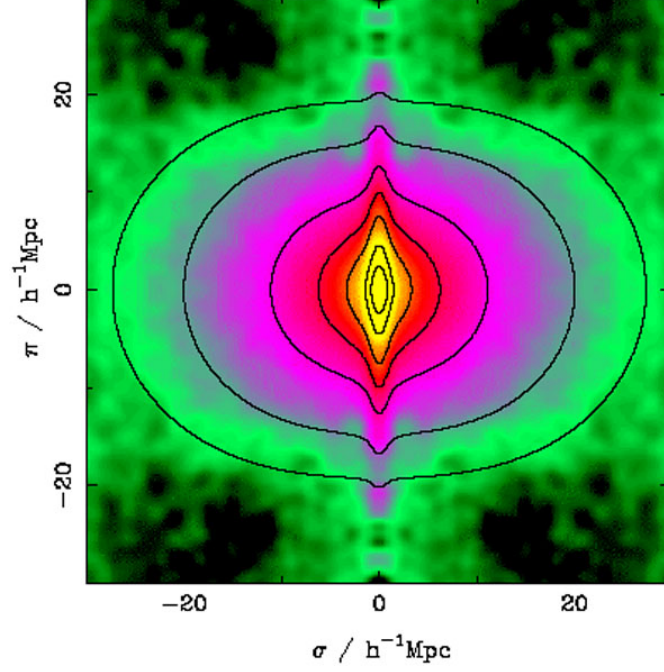


Figure 2.5 – Correlation function of galaxies from the 2dFGRS survey. Galaxy pairs are counted as a function of their radial distance σ and their line-of-sight distance π . Credit: Peacock et al. [2001](#)

Since redshift space is only a deformation of real space, the amount of matter is conserved, and we can write

$$[1 + \delta(\mathbf{x}_{\text{obs}})] d^3 \mathbf{x}_{\text{obs}} = [1 + \delta(\mathbf{x})] d^3 \mathbf{x}_{\text{cos}}, \quad (2.107)$$

and in spherical coordinates

$$[1 + \delta(\mathbf{x}_{\text{obs}})] r_{\text{obs}}^2 dr_{\text{obs}} d\Omega = [1 + \delta(\mathbf{x})] r_{\text{cos}}^2 dr_{\text{cos}} d\Omega, \quad (2.108)$$

where the angular part $d\Omega$ is unchanged since velocities only affect the line of sight. We then obtain

$$1 + \delta(\mathbf{x}_{\text{obs}}) = [1 + \delta(\mathbf{x})] \frac{r_{\text{cos}}^2}{r_{\text{obs}}^2} \frac{dr_{\text{cos}}}{dr_{\text{obs}}} \quad (2.109)$$

$$= [1 + \delta(\mathbf{x})] \left(1 + \frac{v_{\parallel}}{aHr}\right)^{-2} \left(1 + \frac{1}{aH} \frac{\partial}{\partial r_{\text{cos}}} v_{\parallel}\right)^{-1}. \quad (2.110)$$

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At first order we have

$$1 + \delta(\mathbf{x}_{\text{obs}}) = [1 + \delta(\mathbf{x}_{\text{cos}})] \left(1 - 2 \frac{v_{\parallel}}{aHr} \right) \left(1 - \frac{1}{aH} \frac{\partial}{\partial r_{\text{cos}}} v_{\parallel} \right). \quad (2.111)$$

Galaxy power spectrum

The term $v_{\parallel}/aH$ in Eq. 2.111 corresponds to the spatial displacement due to the velocity. It is generally quite small, of order $\sim 10 \text{ Mpc}.h^{-1}$. At large scales, it can be neglected to obtain, again at first order,

$$\delta(\mathbf{x}_{\text{obs}}) = \delta(\mathbf{x}_{\text{cos}}) - \frac{1}{aH} \frac{\partial}{\partial r_{\text{cos}}} v_{\parallel}(\mathbf{x}_{\text{cos}}). \quad (2.112)$$

We can go to Fourier space

$$\delta_{m,\text{obs}}(\mathbf{k}) = \delta_m(\mathbf{k}) - \frac{i}{aH} \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \frac{\partial}{\partial r_{\text{cos}}} v_{\parallel}(\mathbf{x}). \quad (2.113)$$

We can use Eq. 2.70 to obtain

$$\delta_{m,\text{obs}}(\mathbf{k}) = \delta_m(\mathbf{k}) - if \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \frac{\partial}{\partial r_{\text{cos}}} \left[\int \frac{d^3\mathbf{k}'}{(2\pi)^3} \frac{\mathbf{k}' \cdot \hat{\mathbf{x}}}{k'^2} \theta(\mathbf{k}') e^{i\mathbf{k}' \cdot \mathbf{x}} \right] \quad (2.114)$$

$$= \delta_m(\mathbf{k}) + f \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \frac{(\mathbf{k}' \cdot \hat{\mathbf{x}})^2}{k'^2} \theta(\mathbf{k}') e^{i\mathbf{k}' \cdot \mathbf{x}}. \quad (2.115)$$

We often use the approximation called the "plane-parallel" approximation. This approximation assumes that if we work within a sky area of restricted angular size, such that the angular distance between objects is negligible compared to their radial distance, we can write $\hat{\mathbf{x}} \simeq \mathbf{e}_z$. We then have

$$\delta_{m,\text{obs}}(\mathbf{k}) = \delta_m(k) + f \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \frac{(\mathbf{k}' \cdot \mathbf{e}_z)^2}{k'^2} \theta(\mathbf{k}') e^{i\mathbf{k}' \cdot \mathbf{x}} \int d^3\mathbf{x} e^{-i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{x}} \quad (2.116)$$

$$= \delta_m(k) + f \int d^3\mathbf{k}' \frac{(\mathbf{k}' \cdot \mathbf{e}_z)^2}{k'^2} \theta(\mathbf{k}') e^{i\mathbf{k}' \cdot \mathbf{x}} \delta_D(\mathbf{k} - \mathbf{k}') \quad (2.117)$$

$$= \delta_m(k) + f \theta(\mathbf{k}) (\hat{\mathbf{k}} \cdot \mathbf{e}_z)^2. \quad (2.118)$$

Setting $\mu_{\mathbf{k}} = \hat{\mathbf{k}} \cdot \mathbf{e}_z$ we have

$$\delta_{m,\text{obs}}(\mathbf{k}) = \delta_m(\mathbf{k}) + f \mu_{\mathbf{k}}^2 \theta(\mathbf{k}). \quad (2.119)$$

The effect of peculiar velocities is thus to add an anisotropic term to the matter fluctuation field. We will see that it is in fact the measurement of this anisotropy that allows f to be measured indirectly.

The matter distribution is often observed using that of galaxies. These two distributions are not identical, and the relation between them is the subject of study of bias expansion (Desjacques et al. 2018). The simplest relation is to assume a linear bias

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$\delta_g = b\delta_m$, in which case we write

$$\delta_{g,\text{obs}}(\mathbf{k}) = b\delta_m(\mathbf{k}) + f\mu_{\mathbf{k}}^2\theta(\mathbf{k}). \quad (2.120)$$

We then have the power spectrum of the galaxy distribution

$$P_{gg,\text{obs}}(\mathbf{k}) = b^2 P_{\delta\delta}(k) + 2fb\mu_{\mathbf{k}}^2 P_{\theta\delta}(k) + f^2\mu_{\mathbf{k}}^4 P_{\theta\theta}(k). \quad (2.121)$$

We can note that Eq. 2.121 in the linear regime simplifies to

$$P_{gg,\text{obs}}(\mathbf{k}) = b^2 (1 + \beta\mu_{\mathbf{k}}^2)^2 P_{\delta\delta}(\mathbf{k}), \quad (2.122)$$

where $\beta = f/b$. These results remain valid only within the "plane-parallel" approximation; a higher-order expansion is proposed in Lai et al. 2023. The nonlinear part of the distortions is generally modeled by a function F such that

$$P_{gg,\text{obs}}(\mathbf{k}) = b^2 (1 + \beta\mu_{\mathbf{k}}^2)^2 P_{\delta\delta}(\mathbf{k}) F(k\sigma_v\mu_{\mathbf{k}}), \quad (2.123)$$

where σ_v is a nuisance parameter related to the velocity dispersion and must be determined. A common form for F is (Song and Percival 2009)

$$F(k\sigma_v\mu_{\mathbf{k}}) = \frac{1}{1 + (k\sigma_v\mu_{\mathbf{k}})^2}. \quad (2.124)$$

Another parametrization used is (Peacock and Dodds 1994)

$$F(k\sigma_v\mu_{\mathbf{k}}) = e^{-(k\sigma_v\mu_{\mathbf{k}})^2}. \quad (2.125)$$

It should be noted that these two parametrizations are empirical and remain imperfect in describing the nonlinearities (Scoccimarro 2004).

Current results for $f\sigma_8$ and forecasts for future surveys

Redshift-space distortions have been very widely used to measure $f\sigma_8$ up to now. Among existing measurements we can cite those from the surveys WiggleZ (Blake et al. 2011), 6dFGRS (Beutler et al. 2012), SDSS-II (Samushia et al. 2012), SDSS-MGS (Howlett et al. 2015), FastSound (Okumura et al. 2016), VIPERS (Pezzotta et al. 2017; Torre et al. 2017), SDSS-III BOSS (Beutler et al. 2017; Grieb et al. 2017; Sánchez et al. 2017; Satpathy et al. 2017) and SDSS-IV eBOSS (Bautista et al. 2020; Gil-Marín et al. 2020; Mattia et al. 2021; Tamone et al. 2020; Hou et al. 2020; Neveux et al. 2020). These measurements are shown in Fig. 2.6. We observe that the measurements are, for the most part, compatible with the standard GR + Λ CDM model.

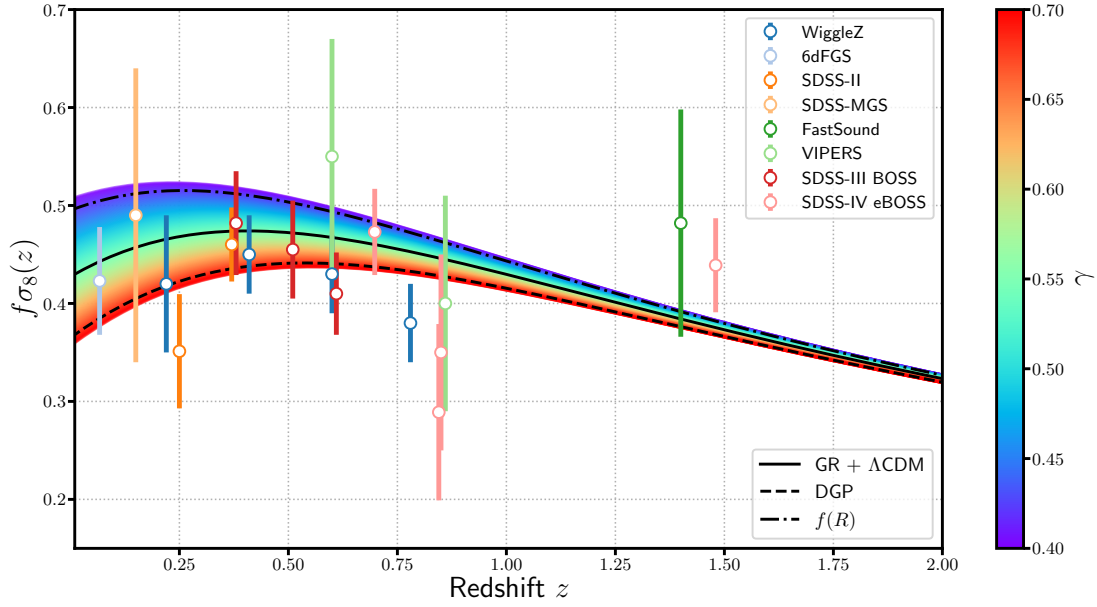


Figure 2.6 – Measurements of $f\sigma_8$ using redshift-space distortions. We show the values of $f\sigma_8$ for different values of γ . The solid line shows the value of $f\sigma_8$ expected for the standard GR + Λ CDM model. The dashed lines show example values of $f\sigma_8$ for the modified gravity models DGP (Linder and Cahn 2007) and $f(R)$ (Tsujikawa et al. 2009).

In Fig. 2.7 we plot the forecasts for RSD measurements from the future surveys DESI (DESI Collaboration et al. 2016) and Euclid (Amendola et al. 2018a).

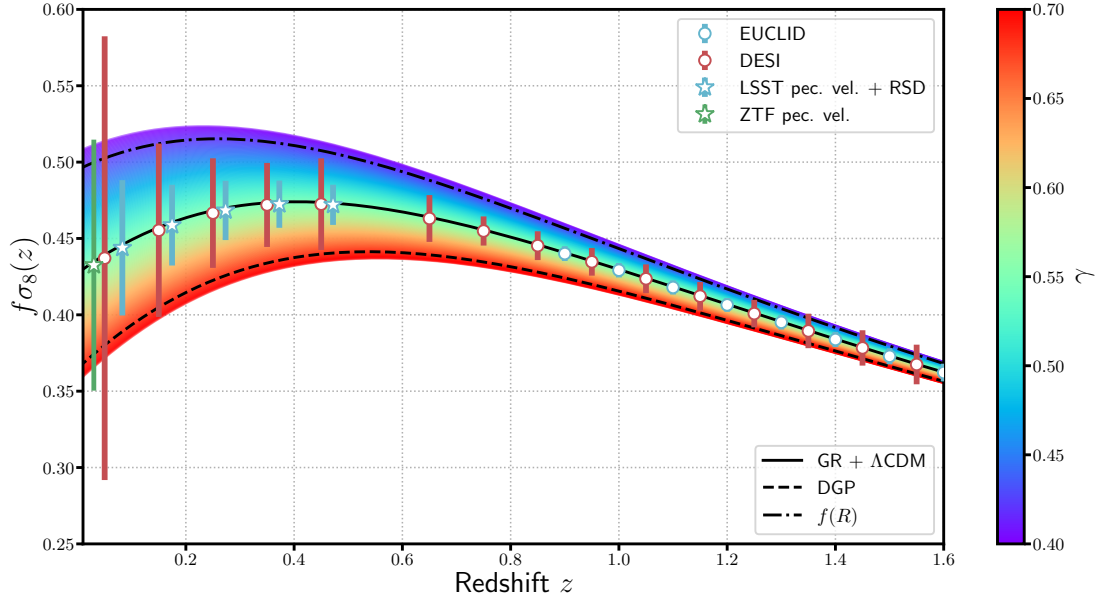


Figure 2.7 – Forecasts for the measurement of $f\sigma_8$ with RSD from the DESI (in green) and Euclid (in red) surveys.

We can note that the precision of RSD measurements at high *redshift* will make it possible to achieve an error on $f\sigma_8$ of order $\sim 1\%$. This precision decreases at low *redshift* because estimating the correlation function requires large statistics and the number of galaxies decreases with *redshift* due to the volume effect. In what follows we will see that at low *redshift* the velocity field can be directly measured using a *redshift* measurement and a distance indicator such as, for example, SNe Ia.

3 Type Ia supernovae: standard candles and direct tracers of the velocity field

Type Ia supernovae, in addition to being remarkable astrophysical phenomena, are among the major probes of cosmology. Their properties enable precise distance measurements that are used to inform us about the content of the Universe through the construction of the Hubble diagram. With the arrival of new surveys, the statistics of SNe Ia reach levels sufficient to open the door to new uses of these probes, such as the study of the velocity field of matter structures. In Sec. 3.1, we present supernovae and their properties. Then, in Sec. 3.2, we explain how a Hubble diagram is built with SNe Ia. In Sec. 3.3, we see how the Hubble diagram is used to constrain cosmology. Finally, in Sec. 3.4, we show how SNe Ia can be used to estimate the velocity field of matter structures.

3.1 Supernovae

Supernovae are explosions of stars at the end of their life. Among all types of supernovae (SNe), those of type Ia (SNe Ia) are among the most luminous. At its peak, the luminosity of a SN Ia is comparable to that of its host galaxy. Moreover, the intrinsic luminosity is observed to vary little from one SN Ia to another. In what follows we will see how the properties of SNe Ia allow distances to be measured and how, once combined with the measurement of the *redshift* of their hosts, SNe Ia are used as precise cosmological probes.

3.1.1 Different types of supernovae

The classification of supernovae is empirical, based on the characteristics of their light spectrum. That of SNe Ia is distinguished by the presence of a silicon absorption line and the absence of hydrogen. These supernovae are caused by the thermonuclear explosion of a white dwarf. They are the most luminous. The other supernovae (Ib, Ic, II) are caused by the gravitational collapse of the star's core (*core collapse*).

3.1.2 Type Ia supernovae

The progenitor stars of SNe Ia are white dwarfs. White dwarfs are very dense stars. The equilibrium of white dwarfs is ensured by the electron degeneracy pressure, which compensates for gravity. In 1931, Subrahmanyan Chandrasekhar showed that there exists a critical mass, $M_{Ch} \sim 1.5 M_{\odot}$, beyond which the degeneracy pressure no longer compensates for gravity. The progressive accretion of matter from a companion star by the white dwarf allows this critical mass to be reached. A thermonuclear explosion then occurs, followed by a deflagration; this is referred to as a supernova. The nature of the companion star is still debated: it may be that the white dwarf accretes matter from a non-degenerate companion star (a red giant, for example), or that the companion star is also a white dwarf and the two merge.

3.1.3 Standard candles

Standard candles are astrophysical objects whose intrinsic luminosity is identical. It is observed that the intrinsic luminosity of SNe Ia is only weakly dispersed around a mean value L . More precisely, we will see in Sec. 3.3.1 that this dispersion can be reduced thanks to the empirical relation between the characteristics of SNe Ia and their intrinsic luminosity; we then speak of standardizable candles. For standard candles, measuring the flux F allows a measurement of the luminosity distance d_L to be inferred,

$$d_L = \sqrt{\frac{L}{4\pi F}}. \quad (3.1)$$

We saw in Sec. 1.1.3 that measuring luminosity distances together with the corresponding *redshifts* allows the history of the expansion of the universe to be retraced. This is represented graphically in a Hubble diagram.

Among supernovae, it appears that only SNe Ia can be used as *standard candles*¹. We therefore want to be able to identify and reject other types of supernovae. Analyzing their spectrum allows an almost error-free identification but is very demanding in telescope time. Classification using photometric light curves is less demanding in observing time but also less reliable.

3.2 Building the Hubble diagram of SNe Ia

The Hubble diagram retraces the history of the expansion of the universe. Each point in the diagram is a *redshift*-distance modulus pair (z, μ) . The *redshift* is measured from the spectrum of the SN Ia's host galaxy. In this section, as well as in the following ones, some properties of SNe Ia and their Hubble diagram are illustrated using preliminary data from the ZTF DR2. The ZTF survey is described in more detail in Chap. 4.

1. Some work seems to show that SNe II can also be standardized (de Jaeger et al. 2020)

3.2.1 Photometry

SNe Ia are observed in several wavelength ranges using a telescope equipped with filters and a camera.

The emission spectrum (or flux density) of a SN Ia $\phi(\lambda, t)$ represents the power emitted per unit wavelength at a time t . It is preferable to use the phase p , given by $p = t - t_0$ with t_0 a reference time, generally taken as the time of peak luminosity. The total luminosity $L(p)$, homogeneous to a power, of a SN Ia is given by

$$L(p) = \int_0^{+\infty} \phi(p, \lambda) d\lambda, \quad (3.2)$$

and the number of photons emitted per unit time is

$$N(p) = \int_0^{+\infty} \phi(p, \lambda) \frac{\lambda}{hc} d\lambda. \quad (3.3)$$

We can write the luminosity in a filter b of transmission $T_b(\lambda)$

$$L_b(p) = \int_0^{+\infty} \phi(p, \lambda) T_b(\lambda) d\lambda, \quad (3.4)$$

and the associated number of photons

$$N_b(p) = \int_0^{+\infty} \phi(p, \lambda) \frac{\lambda}{hc} T_b(\lambda) d\lambda. \quad (3.5)$$

The observed SNe Ia are located at comoving distances of several hundred Mpc from Earth. Because of the expansion of the universe, the observed phase p and wavelength λ , p_{obs} and λ_{obs} , differ from those in the supernova's rest frame p_s and λ_s . We have the relation

$$p_{\text{obs}} = p_s(1 + z) \quad (3.6)$$

$$\lambda_{\text{obs}} = \lambda_s(1 + z), \quad (3.7)$$

where z is the redshift of the SN Ia. The flux received by an observer on Earth through a filter located in their frame is given by

$$F(p_{\text{obs}}; T_b) = \frac{1}{4\pi d_L^2} \int_0^{+\infty} \phi\left(\frac{p_{\text{obs}}}{1+z}, \lambda_s\right) T_b(\lambda_s(1+z)) d\lambda_s, \quad (3.8)$$

and the number of photons received per unit time and area is finally given by

$$S(p_{\text{obs}}; T_b) = \frac{1+z}{4\pi d_L^2} \int_0^{+\infty} \phi\left(\frac{p_{\text{obs}}}{1+z}, \lambda_s\right) \frac{\lambda_s}{hc} T_b(\lambda_s(1+z)) d\lambda_s. \quad (3.9)$$

In practice, measurement devices do not directly provide the flux or photon count. The photons received by the detector are converted into electrons via the photoelectric

effect. After amplification, the resulting signal is expressed in a unit called ADU (*Analog Digital Units*), specific to the instrument. This signal can be obtained by modifying the transmission function T_b so that it accounts for the instrument's transmission and so that the signal amplitude is given in ADU. The curve representing the evolution of the signal as a function of time is called the light curve.

In Fig. 3.1, we show three examples of light curves of SNe Ia observed by ZTF in three filters, g, r and i (see Chap. 4), at different *redshifts*.

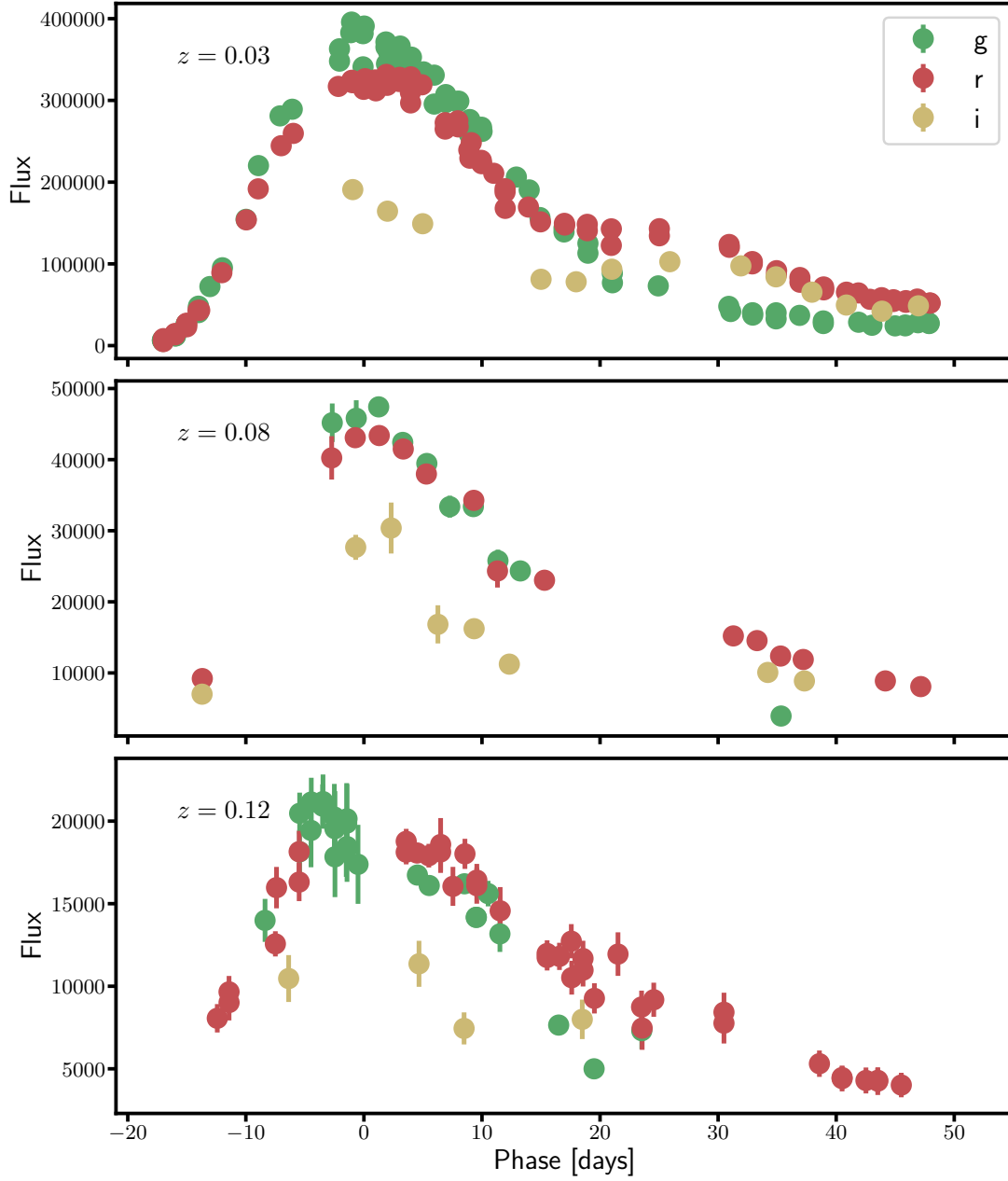


Figure 3.1 – Light curves for three SNe Ia from the ZTF survey DR2 at three different redshifts.

3.2.2 Magnitude system

The bolometric magnitude (i.e., independent of wavelength) corresponding to an observed flux F is

$$m = -2.5 \log_{10} \left(\frac{F}{F_{\text{ref}}} \right), \quad (3.10)$$

where F_{ref} is the flux of a reference source. The quantity called the zero-point of the magnitude system is

$$ZP = 2.5 \log_{10} (F_{\text{ref}}), \quad (3.11)$$

which allows Eq. 3.10 to be rewritten as

$$m = -2.5 \log_{10} (F) + ZP. \quad (3.12)$$

Note that it is the measurement of the zero-point for stars with known photon flux that allows the conversion from the ADU unit system to a photon flux. In a particular band b the magnitude can be written

$$m_b = -2.5 \log_{10} (F_b) + ZP_b. \quad (3.13)$$

where F_b is the flux through this band and ZP_b is computed from the reference flux in the same band.

The magnitude can also be defined using the number of photons received S , correcting for the factor $(1 + z)$

$$m = -2.5 \log_{10} \left(\frac{S}{(1 + z) S_{\text{ref}}} \right) \quad (3.14)$$

where S_{ref} is the number of photons received from a reference source at $z = 0$.

Historically, the star Vega was used as a reference source; today the AB magnitude system is commonly used, based on a source with spectral flux density $\phi_{AB}(\nu) = 3631$ Jy ($1 \text{ Jy} = 10^{-26} \text{ W.m}^{-2}.\text{Hz}^{-1}$).

An important definition for cosmology with SNe Ia is that of the absolute magnitude at 10 pc. It represents the magnitude measured by an observer placed 10 pc from the star.

$$M = -2.5 \log \left(\frac{L}{4\pi(10 \text{ pc})^2} \right) + ZP. \quad (3.15)$$

This magnitude is independent of the cosmological model and depends only on L for SNe Ia considered as standard candles.

3.2.3 The distance modulus

Since SNe Ia are located at different *redshifts*, the photons observed in a telescope filter were not emitted in the same spectral band in the SN Ia's rest frame. We need an observable for which the integral over λ_s is independent of the *redshift*. The flux

received through an arbitrary band b placed in the SN Ia's rest frame is written

$$F_b(p_{\text{obs}}; z) = \frac{1}{4\pi d_L^2} \int_0^{+\infty} \phi\left(\frac{p_{\text{obs}}}{1+z}, \lambda_s\right) T_b(\lambda_s) d\lambda_s \quad (3.16)$$

$$= \frac{L_b\left(\frac{p_{\text{obs}}}{1+z}\right)}{4\pi d_L^2}. \quad (3.17)$$

We can associate the magnitude

$$m_b(p_{\text{obs}}; z) = -2.5 \log_{10} \left(\frac{L_b\left(\frac{p_{\text{obs}}}{1+z}\right)}{4\pi d_L^2} \right) + ZP_b. \quad (3.18)$$

The absolute magnitude at 10 pc of SNe Ia in a band b is given by

$$M_b(p_{\text{obs}}; z) = -2.5 \log_{10} \left(\frac{L_b\left(\frac{p_{\text{obs}}}{1+z}\right)}{4\pi (10 \text{ pc})^2} \right) + ZP_b \quad (3.19)$$

The distance modulus μ is then defined as

$$\mu = m - M \quad (3.20)$$

$$= m_b - M_b \quad (3.21)$$

$$= 5 \log_{10} \left(\frac{d_L}{10 \text{ pc}} \right). \quad (3.22)$$

The distance modulus is independent of the band used and can be expressed analytically within the framework of a cosmological model.

The quantities defined in this section constitute the basic elements for plotting the Hubble diagram.

3.2.4 Simple Hubble diagram

Observing the SN Ia's light curve in several bands allows its magnitude at peak luminosity ($p_{\text{obs}} = 0$) to be inferred in a reference band (by convention the Bessell-B band is used) placed in its rest frame $m_B \equiv m_B(p = 0)$ (In general m_B is measured up to an additive constant, see Sec. 3.3 for details of the method). We can then measure the distance modulus μ up to the constant M_B , which will need to be adjusted (see Sec. 3.3). If, in addition, the *redshift* z of the supernova is known, the theoretical prediction $\mu(z)$ can be compared to the measured value $\mu = m_B - M_B$.

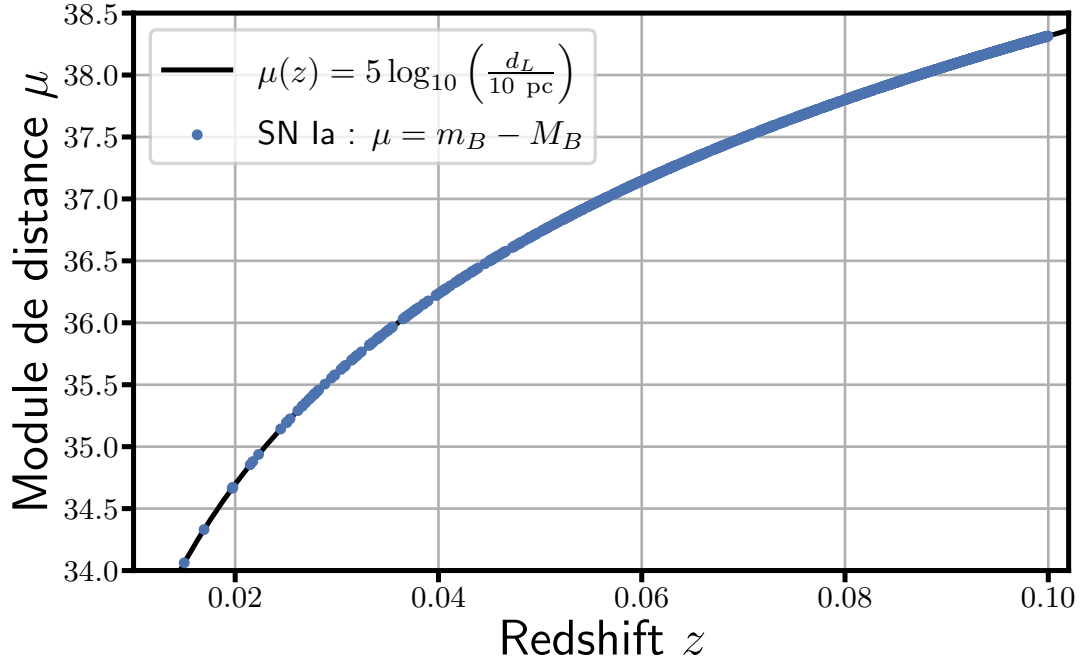


Figure 3.2 – *Simple* Hubble diagram of SNe Ia. In this figure the SNe Ia are considered to be perfectly standard candles.

The *simple* Hubble diagram in Fig. 3.2 is plotted in an ideal case, without measurement errors, and where the SNe Ia are perfectly *standard* candles.

3.3 From data to cosmological constraints

The previous section presented a simple construction of the Hubble diagram. Here, we detail the steps that, starting from the observation of light curves, allow cosmological parameters to be constrained.

3.3.1 Standardization

As indicated in Sec. 3.1.3, there is variability in the luminosity from one SN Ia to another. This dispersion in the luminosities of SNe Ia is of the order of ~ 0.4 mag. In 1993 Phillips showed that there is a correlation between the maximum luminosity of SNe Ia and the decline time of their light curve: the most luminous SNe Ia have a broader light curve (Pskovskii 1977; Phillips 1993). Following these observations, Tripp showed a new correlation between the maximum luminosity and the color measurement of the supernova: the bluest supernovae are the most luminous (Tripp 1998). These correlations (Fig. 3.3) are taken into account to reduce the variations in the absolute magnitudes of SNe Ia by introducing two linear correction parameters in

the Tripp relation

$$M^* = M - \alpha x_1 + \beta c, \quad (3.23)$$

where we can distinguish the parameters common to all SNe Ia: M the absolute magnitude of the population, α the linear correction parameter for the *stretch*, and β the linear correction parameter for the color of the SNe Ia; and the variables specific to each SN Ia: M^* the absolute magnitude of a supernova, x_1 its width parameter² (*stretch*), and c its color.

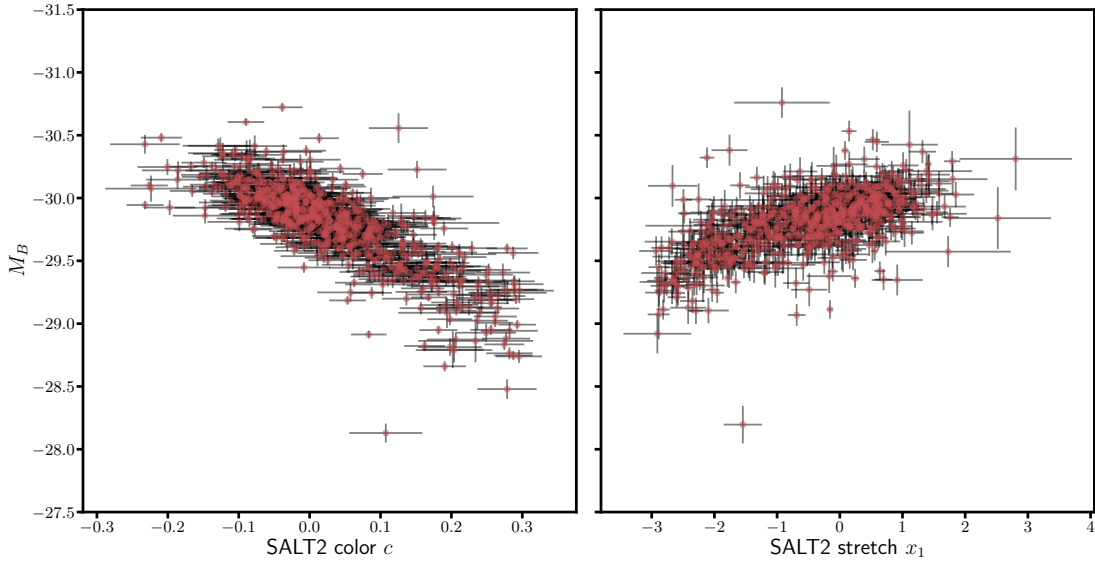


Figure 3.3 – Correlation between the intrinsic luminosity and the color parameter c and width parameter x_1 of the light curve. On the left, we see that the bluer the SN Ia, the brighter it is. On the right, we see that the broader the light curve, the brighter it is.

Several models exist for obtaining the stretch and color parameters of a SN Ia. The model that has been most widely used and tested so far is the SALT2 model (Guy et al. 2007; Guy et al. 2010), which will be detailed in the next section.

Other correlations, between the luminosity and the environment of SNe Ia, are known and are being investigated. For example, a correlation is observed between the magnitude and the host mass (Fig. 3.4) (Sullivan et al. 2010). Recent work compares different environmental observables and their correlation with the magnitude of SNe Ia (Briday et al. 2022).

2. The width parameter has changed over time: Phillips 1993 uses Δm_{15} , which describes the decrease in luminosity of a SN Ia 15 days after its maximum in the Bessell-B band; in Perlmutter et al. 1997 the *stretch* parameter s is introduced, corresponding directly to a coefficient of the temporal width of the light curve, the stretch-corrected phase being $p_s = p/s$; the SALT model of Guy et al. 2005 retains this parameter, while in Guy et al. 2007 the SALT2 model adopts a new parametrization with the coefficient of expansion around the mean spectrum x_1 .

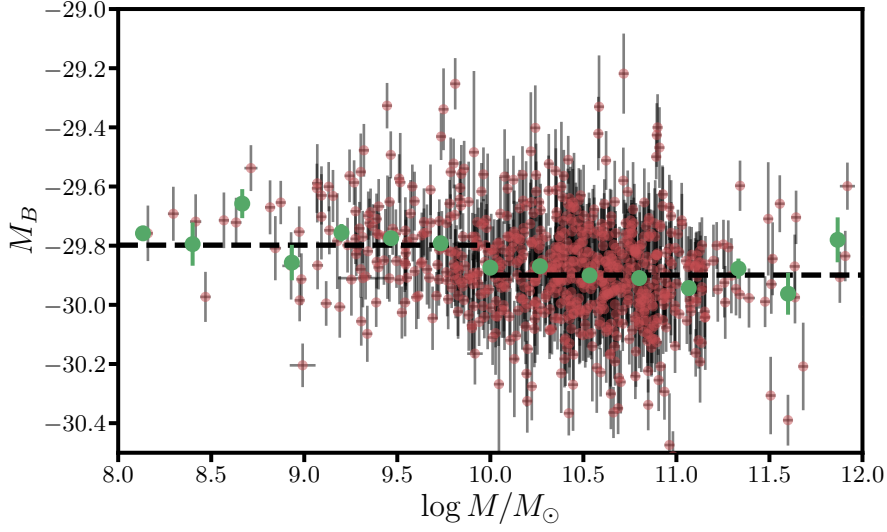


Figure 3.4 – Correlation between the magnitude of SNe Ia and the mass of their host in the ZTF DR2 data.

3.3.2 The SALT2 model

The SALT2 model is a spectro-temporal model for SNe Ia. Its principle relies on a first-order expansion of the flux density of SNe Ia. The flux density ϕ of a SN Ia is expressed as

$$\phi(p_s, \lambda_s) \propto x_0 [M_0(\lambda_s, p_s) + x_1 M_1(\lambda_s, p_s)] e^{cCL(\lambda_s)} \quad (3.24)$$

$$\propto x_0 \phi_{x_1, c}(p_s, \lambda_s), \quad (3.25)$$

where p_s denotes the phase of the light curve relative to the time t_0 of peak luminosity, and λ_s the wavelength. The functions $M_0(p_s, \lambda_s)$ and $M_1(p_s, \lambda_s)$ are, respectively, the mean spectrum and a first-order variation of this spectrum. Both functions depend on the phase (temporal dependence) and the wavelength. The parameter x_1 can then be considered as a first-order coefficient of this expansion. $CL(\lambda_s)$ is an extinction function related to the color c of the SN Ia. Finally, x_0 is a normalization factor describing the amplitude of the spectrum.

The functions M_0 , M_1 and CL are determined during a training process on spectrometric and photometric data. Once these functions are obtained, a parametrized spectro-temporal model for SNe Ia is available, which can be used to obtain, by fitting, the parameters t_0 , x_0 , x_1 and c of any other observational dataset of a SN Ia. Fig. 3.5 shows the model obtained after training in Betoule et al. 2014.

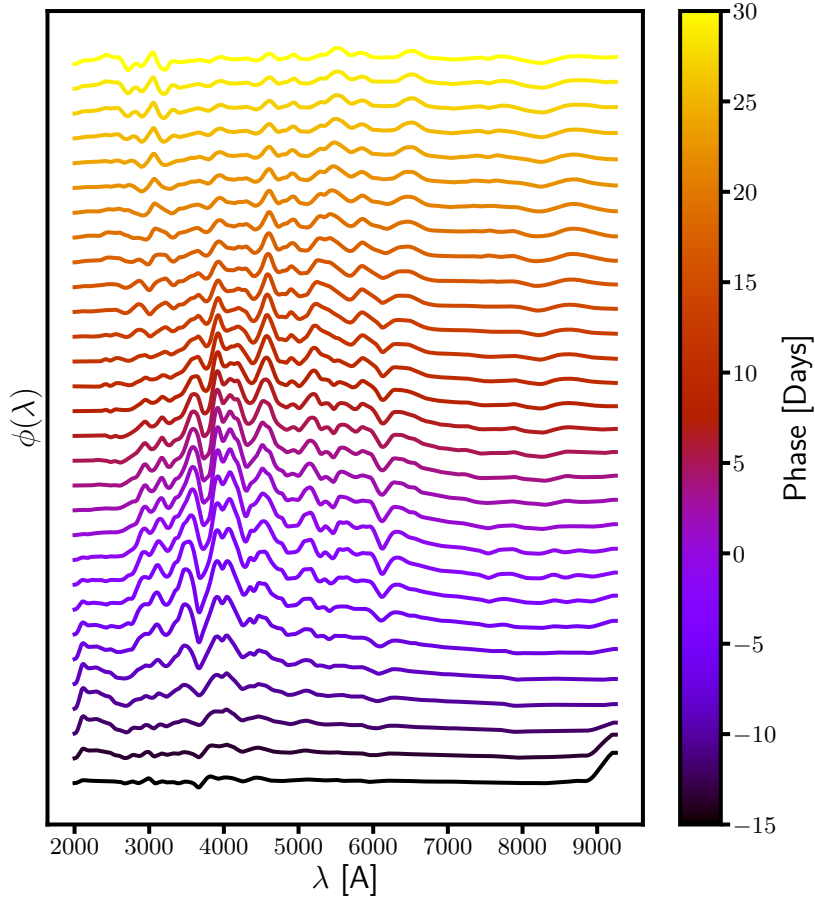


Figure 3.5 – SALT2 spectrum model for $x_1 = 0$ and $c = 0$ for several values of the phase relative to the peak of luminosity.

In practice, the photometric data, i.e., a quantity proportional to the number of photons $S(p_{\text{obs}})$ described in Eq. 3.9, is compared with the model prediction

$$S_{\text{SALT}}(p_{\text{obs}}; T_b) = (1+z)x_0 \int_0^{+\infty} \phi_{x_1,c}(p_s, \lambda_s) T_b(\lambda_s(1+z)) \frac{\lambda_s}{hc} d\lambda_s. \quad (3.26)$$

in order to fit the parameters t_0 , x_0 , x_1 and c . This fit is often performed by minimizing a χ^2

$$\chi_{\text{SALT}}^2 = \sum_i \left(\frac{S_{\text{obs},i} - S_{\text{SALT}}(p_{\text{obs},i}(t_0), T_{b,i}; x_0, x_1, c)}{\sigma_i} \right)^2. \quad (3.27)$$

where $S_{\text{obs},i}$ and σ_i are, respectively, the number of observed photons and the error on this measurement.

Following this fit, the parameters $t_{0,i}$, $x_{0,i}$, $x_{1,i}$ and c_i of each SN Ia are obtained,

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along with the covariance matrix of x_0 , x_1 and c

$$C_{\text{SALT},i} = \begin{pmatrix} \sigma_{x_0,i}^2 & \text{Cov}_{x_0 x_1,i} & \text{Cov}_{x_0 c,i} \\ \text{Cov}_{x_0 x_1,i} & \sigma_{x_1,i}^2 & \text{Cov}_{x_1 c,i} \\ \text{Cov}_{x_0 c,i} & \text{Cov}_{x_1 c,i} & \sigma_{c,i}^2 \end{pmatrix} \quad (3.28)$$

Knowledge of the parameters x_0 , x_1 and c gives us a model proportional to the flux density ϕ of the SN Ia. By identifying Eqs. 3.9 and 3.26, we see that the parameter x_0 contains the cosmological information

$$x_0 \propto d_L^{-2}. \quad (3.29)$$

Using the flux density model given by SALT, we can compute the flux at peak luminosity in the B band of the SN Ia's rest frame

$$F_B = x_0 \int_0^{+\infty} \phi_{x_1,c}(0, \lambda_s) T_B(\lambda_s) d\lambda_s, \quad (3.30)$$

and its magnitude, up to a constant ZP ,

$$m_B = -2.5 \log_{10}(x_0) + ZP. \quad (3.31)$$

It is this last parameter that is used in building the Hubble diagram. To obtain the covariance matrix of m_B , x_1 and c , the covariance matrix is transformed as

$$C_{\text{SALT}}^{(m_B, x_1, c)} = J C_{\text{SALT}}^{(x_0, x_1, c)} J^T. \quad (3.32)$$

where J is the Jacobian matrix such that

$$J = \begin{pmatrix} \frac{\partial m_B}{\partial x_0} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{2.5}{x_0 \ln 10} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3.33)$$

The distance modulus equation 3.21 can be rewritten as

$$\mu = m_B - M_B^* = m_B - (M_B - \alpha x_1 + \beta_c) \quad (3.34)$$

where M_B , α and β are constants common to all SNe Ia that must be determined. Note that knowledge of the constant ZP in Eq. 3.31 is not necessary. As long as this constant is common to all SNe Ia in a dataset, it will be absorbed into the free parameter M_B during the fit of the Hubble diagram (see Sec. 3.3).

The error of the SALT model is propagated for each SN Ia from $(m_{B,i}, x_{1,i}, c_i)$ to μ_i

$$\sigma_{\mu,i}^2 = \mathbf{A} C_{\text{SALT}}^{(m_B, x_1, c)} \mathbf{A}^T, \quad (3.35)$$

where the vector $\mathbf{A}$ is such that

$$\mathbf{A} = \begin{pmatrix} 1 & \alpha & -\beta \end{pmatrix}. \quad (3.36)$$

Correction using the SALT2 model achieves a dispersion of the order of ~ 0.10 mag. Fig. 3.6 shows the dispersion of the ZTF Hubble diagram, before and after standardization.

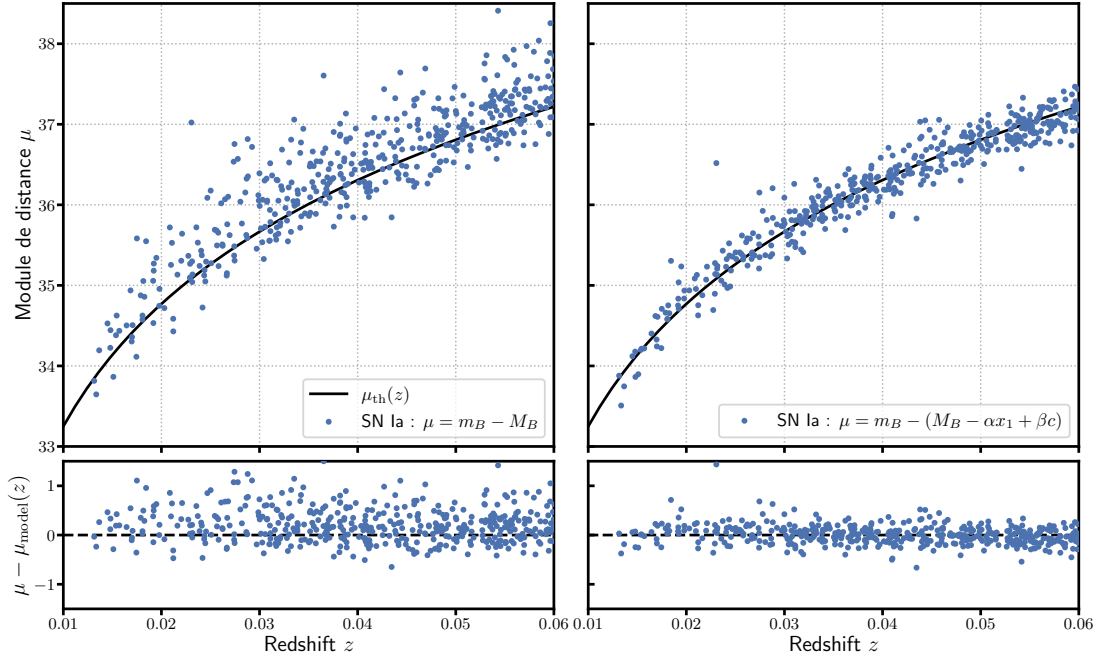


Figure 3.6 – Hubble diagram of SNe Ia from the preliminary ZTF DR2 data before (right) and after (left) standardization.

The dispersion after standardization, called the intrinsic dispersion, can be modeled in several ways. The simplest is to consider it as a random Gaussian variation of the magnitude with standard deviation $\sigma_M \sim 0.10$. More advanced models show that part of the dispersion is wavelength-dependent; the main models are G10 (Guy et al. 2010) and C11 (Chotard et al. 2011).

3.3.3 Cosmological constraints

The SALT2 model therefore allows us, using the measurement of a SN Ia's flux, to model its spectral density with 3 parameters: a magnitude m_B , the *stretch* x_1 and the color c . These parameters are used to estimate the distance modulus μ (Eq. 3.34). However, this modeling contains three free parameters M_B , α and β , unrelated to the cosmological model. These are "standardization" parameters that minimize the dispersion of distances. These parameters can be considered as nuisance parameters

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within a cosmological analysis. We can therefore write, for a SN Ia_{*i*}

$$\mu_i \equiv \mu(m_{B,i}, x_{1,i}, c_i; M_B, \alpha, \beta). \quad (3.37)$$

For a given *redshift* z we have a theoretical expression for the distance modulus μ (Eq. (3.22)). This quantity depends on the cosmological parameters $\mathbf{\Omega}$ through the luminosity distance (Eq. (1.73))

$$\mu_{\text{th},i} = \mu_{\text{th}}(z_i; \mathbf{\Omega}) \quad (3.38)$$

These two expressions can be compared to estimate the cosmological parameters together with the nuisance parameters. One can, for example, minimize a χ^2 (Betoule et al. 2014)

$$\chi^2(\mathbf{\Omega}, M_B, \alpha, \beta) = \mathbf{\Delta\mu}^T C_{\Delta\mu\Delta\mu}^{-1} \mathbf{\Delta\mu}, \quad (3.39)$$

where $\mathbf{\Delta\mu}$ is the data vector of $\Delta\mu_i = \mu_i - \mu_{\text{th},i}$ and $C_{\Delta\mu\Delta\mu}$ the associated covariance matrix. Another method is to explore the likelihood function

$$\mathcal{L}(\mathbf{\Omega}, M_B, \alpha, \beta) = (2\pi)^{-\frac{n}{2}} |C_{\Delta\mu\Delta\mu}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} \chi^2 \right]. \quad (3.40)$$

Fig. 3.7 shows an example of the constraints on the dark energy equation-of-state parameter w and the matter density Ω_m obtained in Betoule et al. 2014. This analysis uses a χ^2 method similar to 3.39.

3 Type Ia supernovae: standard candles and direct tracers of the velocity field – 3.4 Estimating velocities using the Hubble diagram of SNe Ia

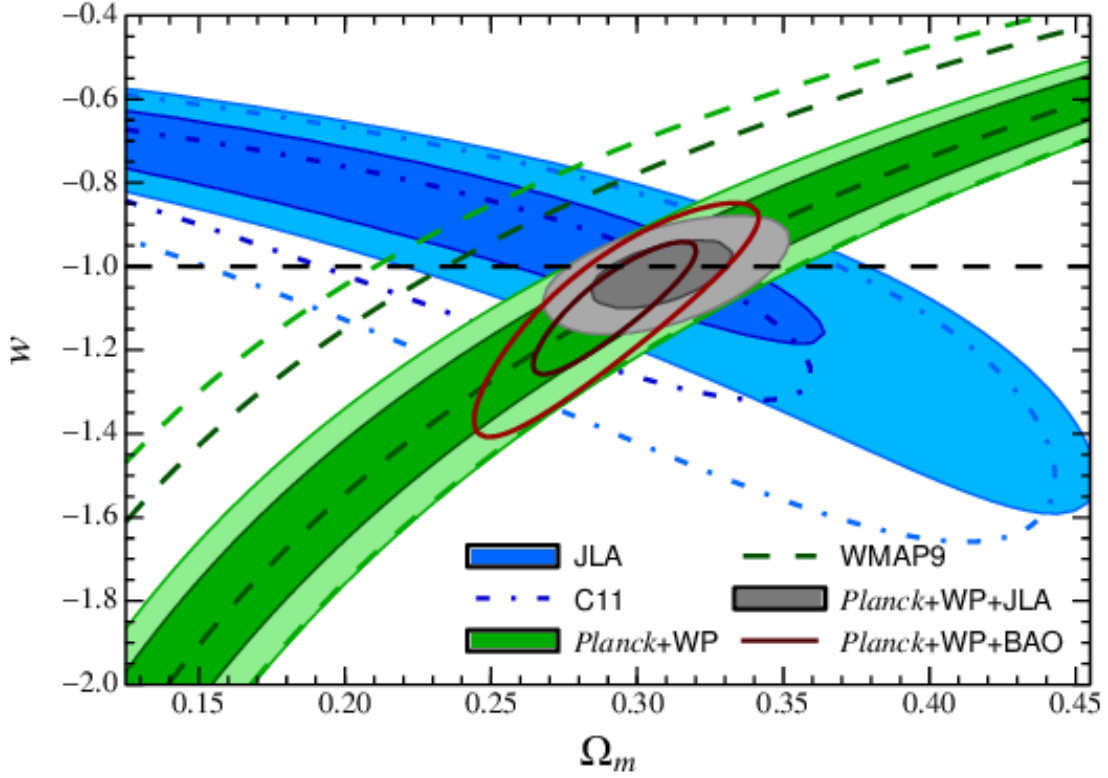


Figure 3.7 – Constraint on the dark energy equation-of-state and matter density parameters w - Ω_m , taken from Betoule et al. 2014

3.4 Estimating velocities using the Hubble diagram of SNe Ia

In Sec. 2.2.1 we saw that peculiar velocities modify the observed *redshift* via the relativistic Doppler effect as well as, to a lesser extent, the luminosity distance. If we neglect the observer's velocity, the observed redshift and distance modulus are given by

$$1 + z_{\text{obs}} = (1 + z_{\text{cos}})(1 + z_p), \quad (3.41)$$

$$\mu_{\text{obs}} = \mu_{\text{cos}} + 10 \log_{10}(1 + z_p). \quad (3.42)$$

In Fig. 3.8 we show these two effects on a Hubble diagram of SNe Ia. The SNe Ia in galaxies moving toward us are shifted to the left of the diagram, while those in galaxies moving away from us are shifted to the right.

3 Type Ia supernovae: standard candles and direct tracers of the velocity field – 3.4 Estimating velocities using the Hubble diagram of SNe Ia

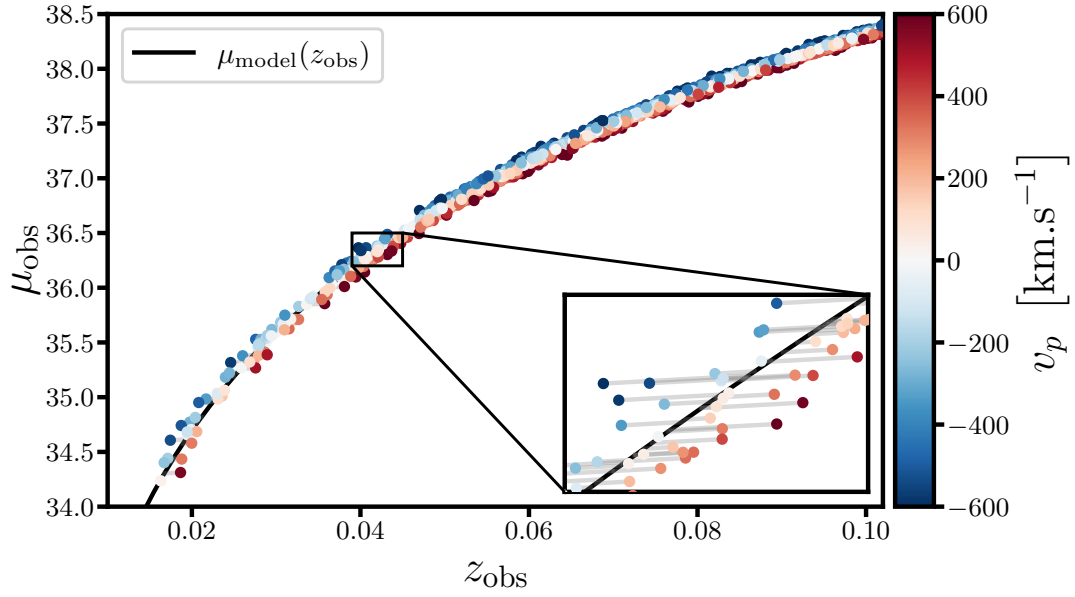


Figure 3.8 – Hubble diagram of SNe Ia with the effects of peculiar velocities.

In Fig. 3.9 we have schematized the different effects. If the SNe Ia were perfectly standard and had no peculiar velocity, we would observe the point $(z_{\text{cos}}, \mu_{\text{cos}})$. The effect of the Doppler shift is plotted in red, leading to the redshift z_{obs} . Combined with the small relativistic aberration effect, shown in yellow, we obtain the observed point $(z_{\text{obs}}, \mu_{\text{obs}})$. The quantity accessible with the Hubble diagram is the residual, which is the difference between the observed distance modulus and the distance modulus of our theoretical cosmological model evaluated at z_{obs} ; this quantity is plotted in green.

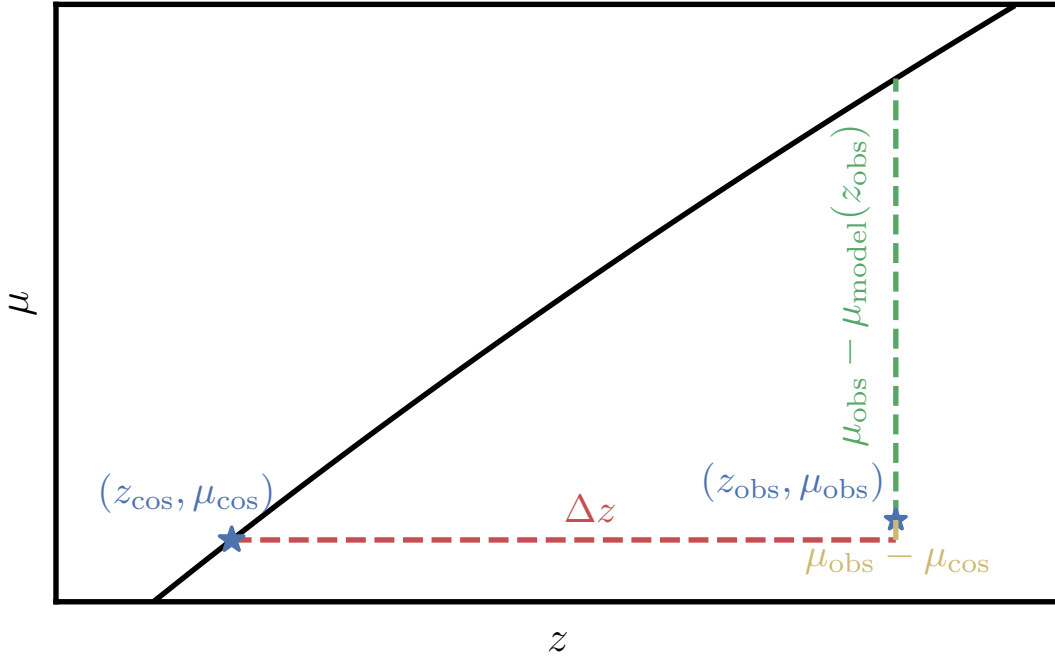


Figure 3.9 – Diagram of the different effects of peculiar velocities on a point of the Hubble diagram. The Doppler shift is plotted in red, the relativistic aberration effect in yellow. The Hubble diagram residual that we can observe is shown in green.

3.4.1 Relation between velocities and Hubble diagram residuals

We can find a formula to quantify the effect of peculiar velocities on the Hubble diagram residuals. The residuals are given by

$$\Delta\mu = \mu_{\text{obs}} - \mu_{\text{th}}(z_{\text{obs}}) = 5 \log_{10} \left(\frac{d_{L,\text{obs}}}{d_{L,\text{th}}(z_{\text{obs}})} \right). \quad (3.43)$$

We consider the case where $d_{L,\text{obs}}$ differs from $d_{L,\text{th}}$ only through the effect of the peculiar velocity. The observed luminosity distance $d_{L,\text{obs}}$ appearing in Eq. 3.43 is then modeled as a function of the peculiar velocity and the cosmological model by

$$d_{L,\text{obs}} \equiv (1 + z_p)^2 d_{L,\text{th}}(z_{\text{cos}}). \quad (3.44)$$

The theoretical luminosity distance in Eq. 3.43 is evaluated at $z \equiv z_{\text{obs}}$, i.e., we have $z = (1 + z_{\text{cos}})(1 + z_p) - 1$. Since the effect of velocities is fairly weak given our current precision on the distance modulus measurement, we use a first-order Taylor series expansion in z_p of $d_{L,\text{th}}$. This expansion assumes that we are in the regime $z_p \ll z_{\text{cos}}$,

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giving

$$d_{L,\text{th}}(z_{\text{obs}}) \simeq d_{L,\text{th}}(z_{\text{cos}}) + \left. \frac{\partial z}{\partial z_p} \frac{\partial d_{L,\text{th}}}{\partial z} \right|_{z_p=0} z_p \quad (3.45)$$

$$\simeq d_{L,\text{th}}(z_{\text{cos}}) + (1 + z_{\text{cos}}) \frac{\partial}{\partial z} \left[(1 + z) \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')} \right] \Big|_{z_p=0} z_p \quad (3.46)$$

$$\simeq d_{L,\text{th}}(z_{\text{cos}}) + (1 + z_{\text{cos}}) \left(\frac{c}{H_0} \int_0^{z_{\text{cos}}} \frac{dz'}{E(z')} + (1 + z_{\text{cos}}) \frac{c}{H_0 E(z_{\text{cos}})} \right) z_p \quad (3.47)$$

$$\simeq d_{L,\text{th}}(z_{\text{cos}}) + \left(d_{L,\text{th}}(z_{\text{cos}}) + \frac{c(1 + z_{\text{cos}})^2}{H(z_{\text{cos}})} \right) z_p \quad (3.48)$$

$$\simeq d_{L,\text{th}}(z_{\text{cos}}) \left[1 + \left(1 + \frac{c(1 + z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} \right) z_p \right] \quad (3.49)$$

Combining Eqs. 3.44 and 3.49 gives

$$d_{L,\text{th}}(z_{\text{obs}}) \simeq \frac{d_{L,\text{obs}}}{(1 + z_p)^2} \left[1 + \left(1 + \frac{c(1 + z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} \right) z_p \right] \quad (3.50)$$

$$\simeq d_{L,\text{obs}} \left[(1 - 2z_p) + \left(1 + \frac{c(1 + z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} \right) z_p \right] \quad (3.51)$$

$$\simeq d_{L,\text{obs}} \left[1 + \left(\frac{c(1 + z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} - 1 \right) z_p \right]. \quad (3.52)$$

This equation can be inverted to obtain

$$d_{L,\text{obs}} \simeq d_{L,\text{th}}(z_{\text{obs}}) \left[1 + \left(\frac{c(1 + z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} - 1 \right) z_p \right]^{-1} \quad (3.53)$$

$$\simeq d_{L,\text{th}}(z_{\text{obs}}) \left[1 + \left(1 - \frac{c(1 + z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} \right) z_p \right] \quad (3.54)$$

We thus find the relative variation in luminosity distance

$$\delta d_L = \frac{d_{L,\text{obs}} - d_{L,\text{th}}(z_{\text{obs}})}{d_{L,\text{th}}(z_{\text{obs}})} \simeq \left(1 - \frac{c(1 + z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} \right) z_p. \quad (3.55)$$

A Taylor expansion of (3.43) in δd_L gives us

$$\Delta\mu = \frac{5}{\ln(10)} \ln(1 + \delta d_L) \quad (3.56)$$

$$\simeq \frac{5}{\ln(10)} \delta d_L. \quad (3.57)$$

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Substituting δd_L in Eq. 3.57 with its approximation given in Eq. 3.55, we finally obtain an expression for $v_p \simeq cz_p$

$$v_p \simeq -\frac{\ln(10)c}{5} \left(\frac{c(1+z_{\text{cos}})}{r(z_{\text{cos}})H(z_{\text{cos}})} - 1 \right)^{-1} \Delta\mu. \quad (3.58)$$

Validity of the different approximations

In Fig. 3.10 we compare δd_L with its Taylor approximation denoted $\widetilde{\delta d_L}$ as a function of redshift z_{cos} . We see that for redshifts $z_{\text{cos}} > 0.01$ the difference between these two quantities is below 10%.

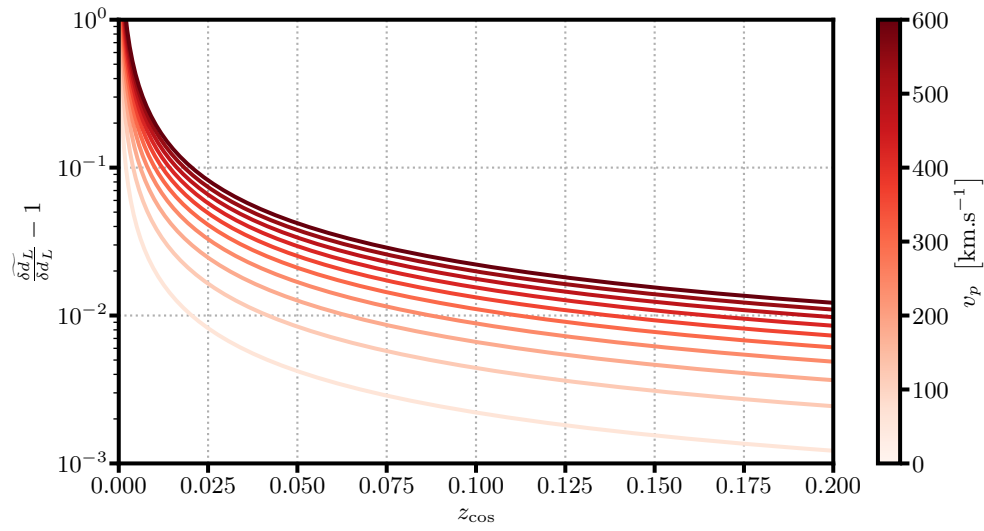


Figure 3.10 – Relative difference of δd_L from its Taylor approximation for different peculiar velocities

Fig. 3.11 shows the value of δd_L as a function of redshift z_{cos} . For $z_{\text{cos}} > 0.01$, $|\delta d_L| < 0.1 \ll 1$.

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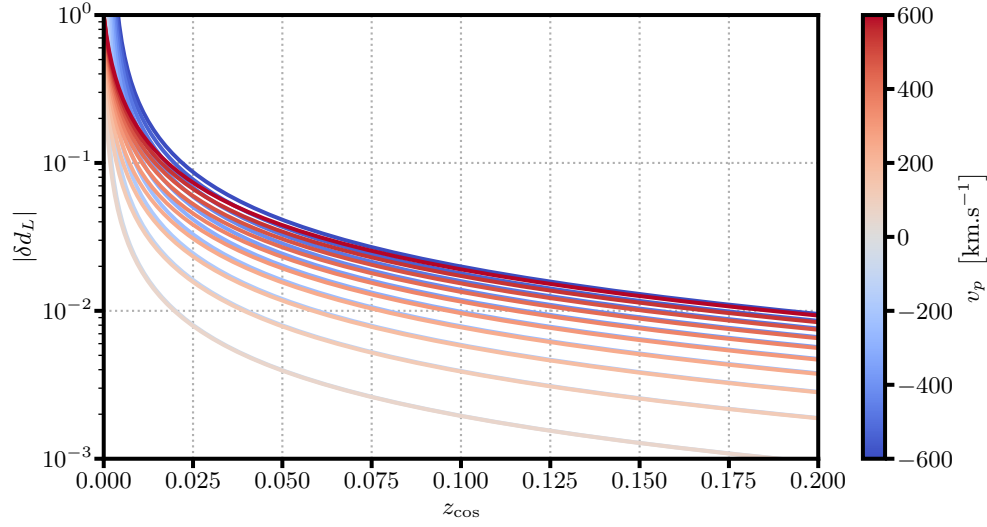


Figure 3.11 – δd_L as a function of redshift for different peculiar velocities

We can see in Fig. 3.12 that the difference between the Taylor expansion $\widetilde{\Delta\mu}$ and the true value of $\Delta\mu$ is below 0.01 mag for a peculiar velocity of 300 km/s.

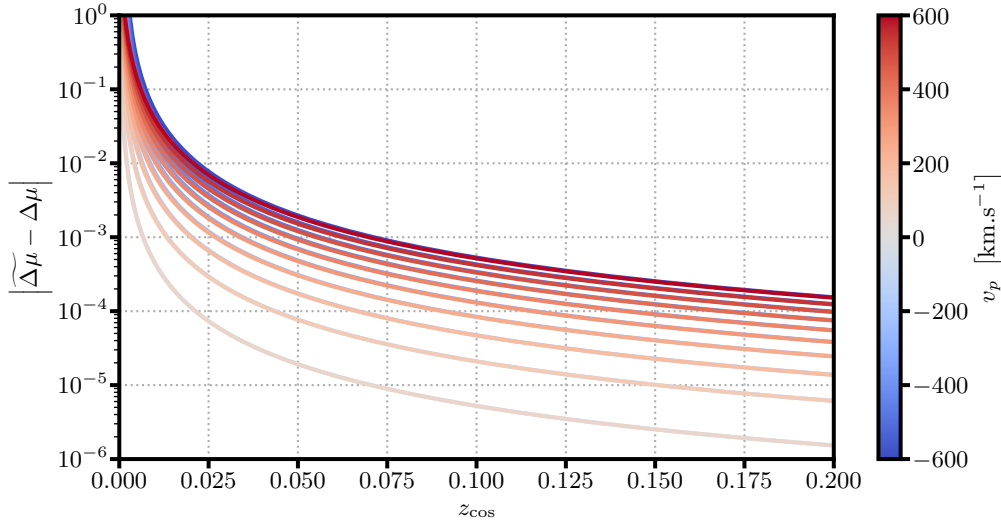


Figure 3.12 – Difference between $\Delta\mu$ and its Taylor expansion $\widetilde{\Delta\mu}$ as a function of redshift for a peculiar velocity $v_p = 300$ km/s

3.4.2 Velocities as a systematic effect in the Hubble diagram

In "classical" cosmological analyses of SNe Ia, peculiar velocities are regarded as a systematic effect to be reduced. In Betoule et al. 2014 the error due to peculiar

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velocities is approximated by

$$\sigma_{pv} = \frac{5}{\ln 10} \frac{cz_p}{z}, \quad (3.59)$$

with $cz_p = 150 \text{ km.s}^{-1}$. This expression corresponds to a $z \rightarrow 0$ approximation of Eq. 3.58. In this analysis the impact of peculiar velocities appears to be negligible compared to other systematics (Table 11 of Betoule et al. 2014). In the Pantheon analysis of Scolnic et al. 2018, peculiar velocities represent $\sim 1.5\%$ of the systematic variance (Table 9 of Scolnic et al. 2018). A study of the impact of peculiar velocities on low-*redshift* SNe Ia by Peterson et al. 2022 showed that using external catalogs to correct velocities could reduce the standard deviation of the Hubble diagram residuals.

3.4.3 Estimating velocities for cosmology

In the framework of an analysis of the growth rate of structure $f\sigma_8$, velocities are no longer a source of systematic error. Instead, we seek to estimate them using the Hubble diagram residuals. In Eq. 3.58, the cosmological redshift z_{cos} appears. This redshift is never directly accessible in observations. Our estimator is therefore built using the observed redshift z_{obs}

$$\hat{v}_p^{(1)} = -\frac{\ln(10)c}{5} \left(\frac{c(1+z_{\text{obs}})}{r(z_{\text{obs}})H(z_{\text{obs}})} - 1 \right)^{-1} \Delta\mu \quad (3.60)$$

However, as proposed in Hui and Greene 2006, using z_{obs} instead of z_{cos} is a second-order approximation. The estimator (3.60) is used in Johnson et al. 2014. In the literature, other versions of the estimator in Eq. 3.60 can be found. Two different approximations can be made; the first is a Taylor expansion of $r(z)H(z)$ at low redshift

$$r(z)H(z) \simeq cz_{\text{mod}}. \quad (3.61)$$

At first order

$$cz_{\text{mod}} = cz, \quad (3.62)$$

and at second order

$$cz_{\text{mod}} = cz \left\{ 1 + \frac{1}{2} (1 - q_0) z - \frac{1}{6} (1 - q_0 - 3q_0^2 + j_0) z^2 \right\}, \quad (3.63)$$

where q_0 and j_0 are, respectively, the deceleration and *jerk* parameters. Using the first-order expansion gives the estimator

$$\hat{v}_p^{(2)} = -\frac{\ln(10)c}{5} z_{\text{obs}} \Delta\mu. \quad (3.64)$$

The second approximation that can be made is to neglect the "-1" term. This approximation is possible because at low z the term $\frac{c(1+z_{\text{obs}})}{r(z_{\text{obs}})H(z_{\text{obs}})} \gg 1$. The estimator is

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then written

$$\hat{v}_p^{(3)} = -\frac{\ln(10)}{5} \frac{r(z_{\text{obs}})H(z_{\text{obs}})}{1+z_{\text{obs}}} \Delta\mu \quad (3.65)$$

This estimator is used in Howlett et al. 2017a. Combining the approximations of Eqs. 3.64 and 3.65 gives

$$\hat{v}_p^{(4)} = -\frac{\ln(10)c}{5} \frac{z_{\text{obs}}}{1+z_{\text{obs}}} \Delta\mu. \quad (3.66)$$

This last estimator is the one presented in Watkins and Feldman 2015 and used in Howlett et al. 2017a. If we write the estimators in the general form

$$\hat{v}_i = -\frac{\ln(10)c}{5} J_i(z_{\text{obs}}) \Delta\mu, \quad (3.67)$$

with

$$\begin{cases} J_1(z) &= \left(\frac{c(1+z)}{r(z)H(z)} - 1 \right)^{-1} \\ J_2(z) &= z \\ J_3(z) &= \frac{r(z)H(z)}{c(1+z)} \\ J_4(z) &= \frac{z}{1+z} \end{cases}. \quad (3.68)$$

And if, moreover, we only consider the error on μ_{obs} , we can express the error on the velocity estimate

$$\sigma_{\hat{v}_i} = \frac{\ln(10)c}{5} J_i(z_{\text{obs}}) \sigma_{\mu}. \quad (3.69)$$

Bias of the estimators

All the estimators derive from the same first-order Taylor expansion, valid under the assumption $z_p \ll z_{\text{cos}}$. They will therefore be more or less biased depending on the velocity and *redshift* values for which they are used. In Fig. 3.13, we compare the biases of the different estimators. We observe that the least biased estimator is $\hat{v}_p^{(1)}$. The approximations tend to lead to an underestimation of all velocities as the redshift increases. This effect is more pronounced when the "-1" term is neglected.

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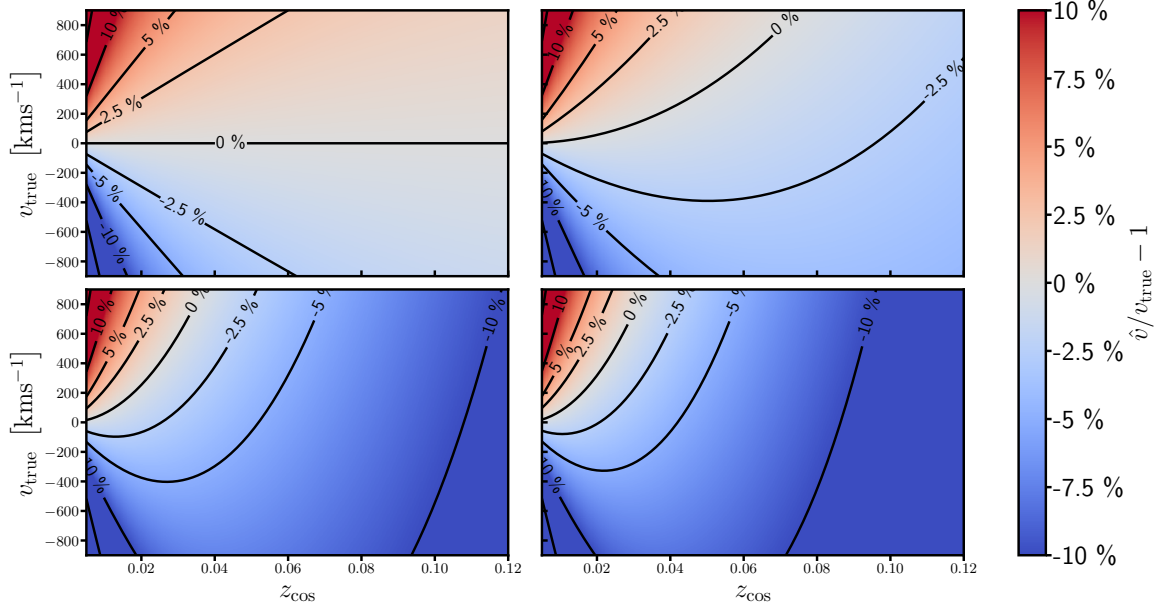


Figure 3.13 – Comparison of the different estimator biases. Top left: $\hat{v}_p^{(1)}$, top right: $\hat{v}_p^{(2)}$, bottom left: $\hat{v}_p^{(3)}$, and bottom right: $\hat{v}_p^{(4)}$.

In this thesis work it is the estimator $\hat{v}_1$ that will be used. However, this requires assuming a fiducial cosmology. In Fig. 3.14, we show the bias of the estimator $\hat{v}_1$ as a function of *redshift* and of the bias on Ω_m . At low *redshift* the bias is always dominated by the error from the Taylor expansion used to derive the estimator. The bias due to an incorrect fiducial cosmology becomes more significant as the *redshift* increases; at a *redshift* $z_{\text{cos}} > 0.1$, an error of $\sim 2\%$ on Ω_m results in a bias on the velocity of $\sim 5\%$. Below a *redshift* of $z \sim 0.06$ the bias is below $\sim 2.5\%$ for an error on Ω_m smaller than $\sim 5\%$.

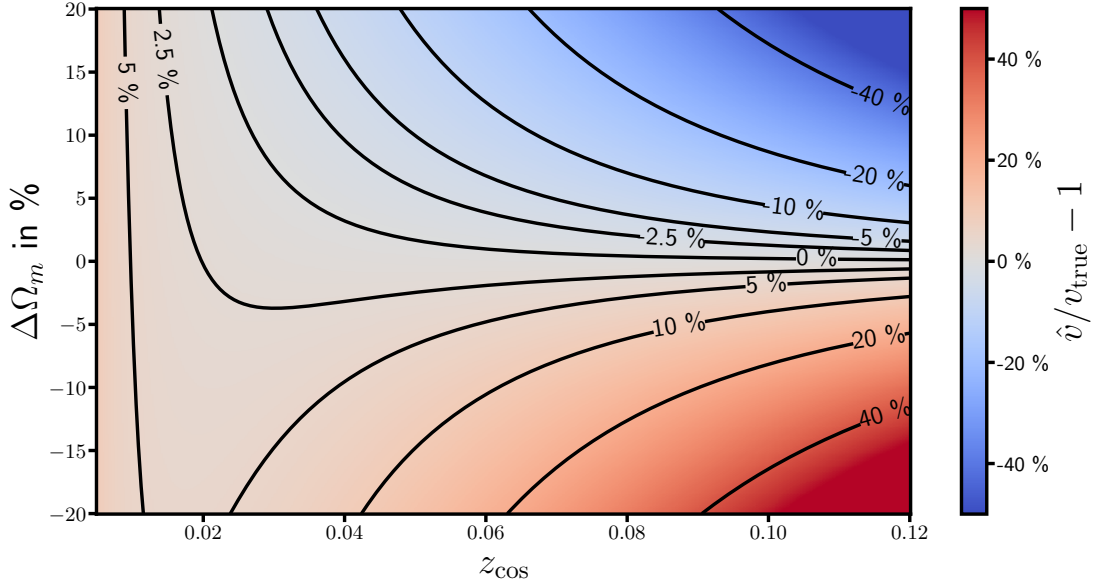


Figure 3.14 – Variation of the velocity estimator as a function of *redshift* z_{cos} and of the bias on the matter density Ω_m . The velocity is fixed at $v_p = 300 \text{ km.s}^{-1}$.

Gaussianity of velocity errors

A widely used assumption in cosmology is that of Gaussian errors. In Chap. 6 we will see that the method used to constrain the growth rate of structure $f\sigma_8$ also requires this assumption. We therefore want to test whether the Gaussianity of the errors on the Hubble diagram residuals $\Delta\mu$ is preserved by our velocity estimator $\hat{v}_1$. To do this, we simulate a dataset of distance moduli μ with peculiar velocity. We proceed as follows

1. We draw N *redshifts* z according to the probability density function $\mathcal{P}(z) = \frac{3r^2(z)}{V(z_{\text{max}}) - V(z_{\text{min}})}$.
2. We draw N velocities v_p according to a normal distribution $\mathcal{N}(0, \sigma_v)$ with $\sigma_v = 300 \text{ km.s}^{-1}$.
3. We draw N Gaussian noise values σ_μ according to a normal distribution $\mathcal{N}(0, \sigma_M)$ with $\sigma_M = 0.12$.
4. We use Eqs. 2.101 and 3.42 to obtain, respectively, z_{obs} and μ_{obs} .

With the simulation thus obtained, we can estimate the velocity and its error. In Fig. 3.15 we plot the quantity $X = (\hat{v}_1 - v_{\text{true}})/\sigma_{\hat{v}}$ for two datasets of $N = 10^6$ distance moduli. The first dataset is in a *redshift* range $z \in [0., 0.02]$ and the second in the range $z \in [0.02, 0.06]$. If the estimator preserves the Gaussian nature of the errors, the histogram of X should follow a standard normal distribution $\mathcal{N}(0, 1)$. We can see in Fig. 3.15 that this is the case for the range $z \in [0.02, 0.06]$. For the lower-*redshift* range we observe a deviation from the Gaussian distribution, likely related to the breakdown of the estimator's approximations in this regime. This non-Gaussianity,

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although small, could lead to an overestimation of the velocity variance. For SNe Ia the amount of data expected at $z < 0.02$ is very small and should not have a strong impact. This issue can nevertheless be avoided by choosing to use only SNe Ia with a *redshift* $z > 0.02$.

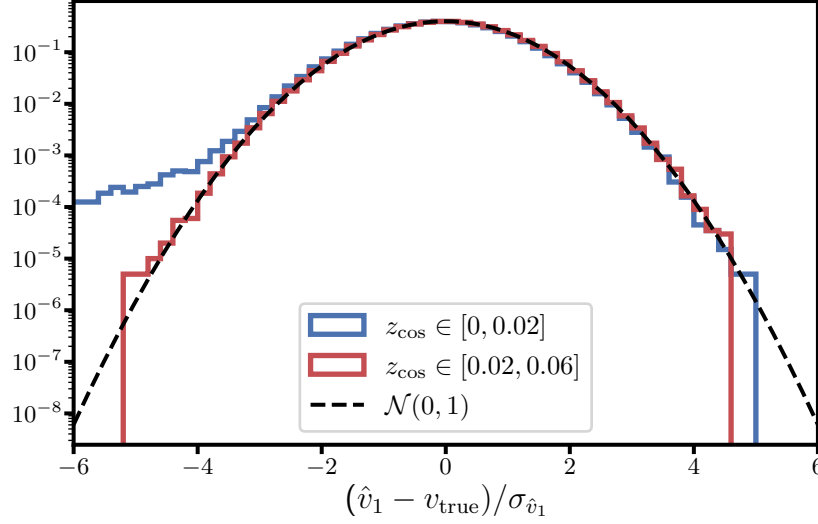


Figure 3.15 – Distribution of $X = (\hat{v}_1 - v_{\text{true}}) / \sigma_{\hat{v}_1}$ for two simulations. In blue: *redshift* range $0 < z < 0.02$. In red: *redshift* range $0.02 < z < 0.06$. At low *redshift* the distribution is no longer Gaussian.

4 Observing SNe Ia with the ZTF survey

This thesis work took place within the collaboration of the *Zwicky Transient Facility* (ZTF) survey. This collaboration uses observations from the *Samuel Oschin* telescope, in California, to carry out research in a variety of fields ranging from the study of astrophysical objects to cosmology. Within the cosmology group, the work focuses on SNe Ia. Indeed, as we will see, ZTF is the first survey to produce such a large dataset of SNe Ia at low *redshift*. In Sec. 4.1 we summarize previous SNe Ia surveys. The ZTF survey is described in Sec. 4.2. Finally, in Sec. 4.3 we discuss the synergies with other surveys, which are necessary for ZTF to be able to provide precise constraints on cosmological parameters.

4.1 Previous SNe Ia surveys

At the time of the discovery of the acceleration of the expansion of the Universe in the late 1990s, Riess et al. 1998 and Perlmutter et al. 1999 had only about forty SNe Ia available. In 2014 the JLA analysis (Betoule et al. 2014) had ~ 740 SNe Ia available. Today, the largest SNe Ia dataset used for cosmological measurements is the Pantheon+ compilation (Scolnic et al. 2022). This compilation gathers 1550 SNe Ia coming from 18 different surveys. This data spans a wide *redshift* range from $z \sim 0.0020$ to $z \sim 2.26$. This compilation makes it possible to place constraints on the density parameters and on the equation of state of dark energy. However, it has a few weaknesses: it requires cross-calibration work between the different surveys, and it is neither homogeneous in *redshift* nor in spatial distribution (Fig. 4.1).

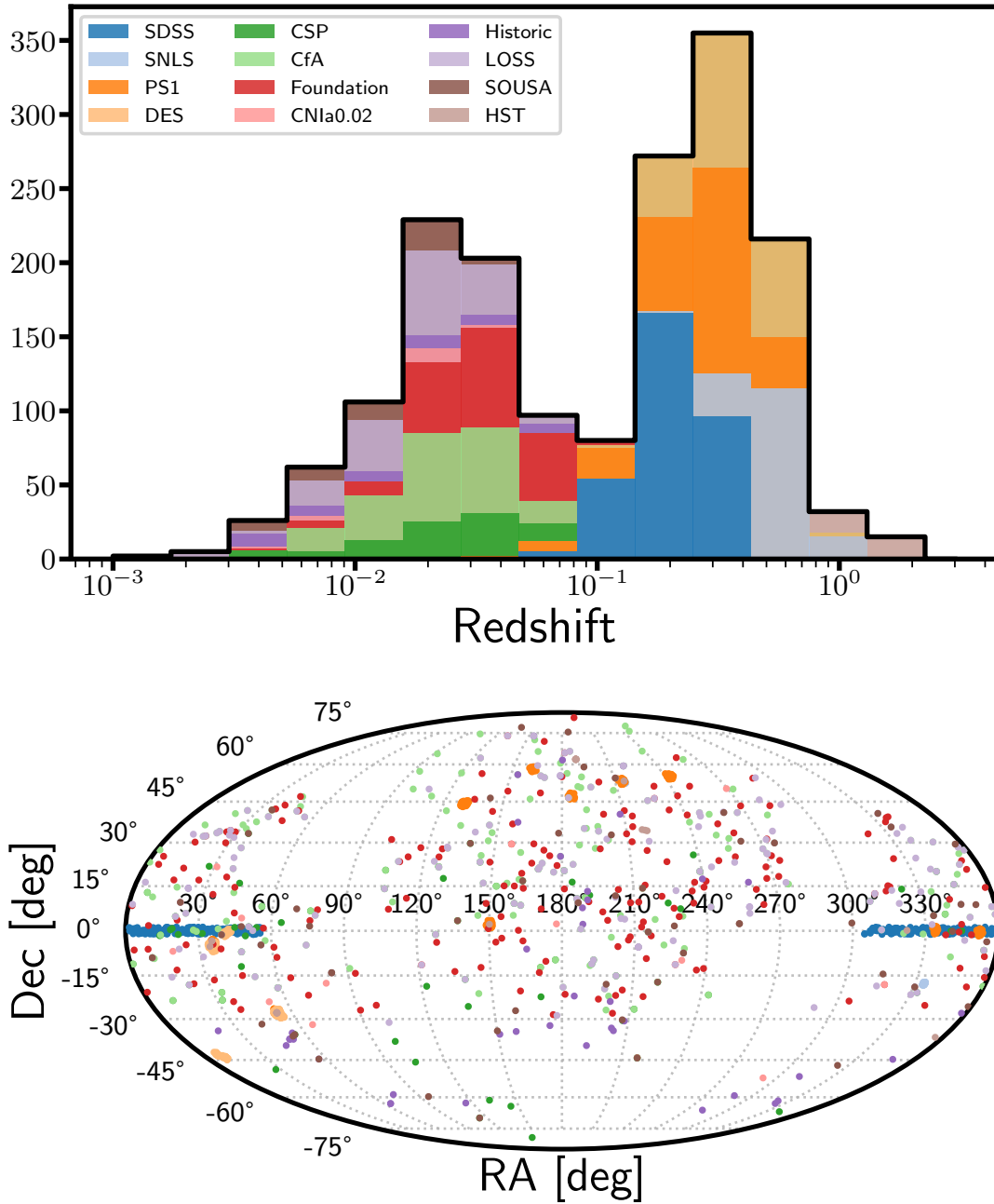


Figure 4.1 – Distribution of *redshifts* (top) and angular distribution (bottom) of the SNe Ia in the Pantheon+ compilation. Adapted from Scolnic et al. [2022](#).

Note that the statistics of SNe Ia at low *redshift* remain rather limited, with ~ 740 SNe Ia below $z \sim 0.1$. This point is of particular interest for analyses involving peculiar velocities, since it is at low *redshift* that their effect is most visible and therefore easiest to estimate.

4.2 The ZTF survey

The ZTF survey is a photometric survey that uses the *Samuel Oschin* telescope at the Palomar Observatory in California (Fig. 4.2). The survey is designed for the study of transient astrophysical objects. We present here a few technical aspects of this survey.



Figure 4.2 – The Samuel Oschin telescope at Palomar Observatory, California. Credits: *Caltech/Palomar*

4.2.1 The ZTF photometric system

The ZTF observing system is described in detail in Dekany et al. 2020. The Samuel Oschin telescope has a mirror with a 1.2 meter radius that directs the photons toward the focal plane where the camera is located. The ZTF camera (Fig. 4.3) is the major upgrade brought to the telescope for this survey. This camera has 16 CCDs of 6144×6160 pixels with a resolution of $\sim 1''$. The CCDs are subdivided into 4 quadrants, each equipped with an amplifier with a gain of $G \simeq 6 e^-/\text{ADU}$. Thus, for each ZTF observation one image per CCD quadrant is obtained, i.e. 64 images.

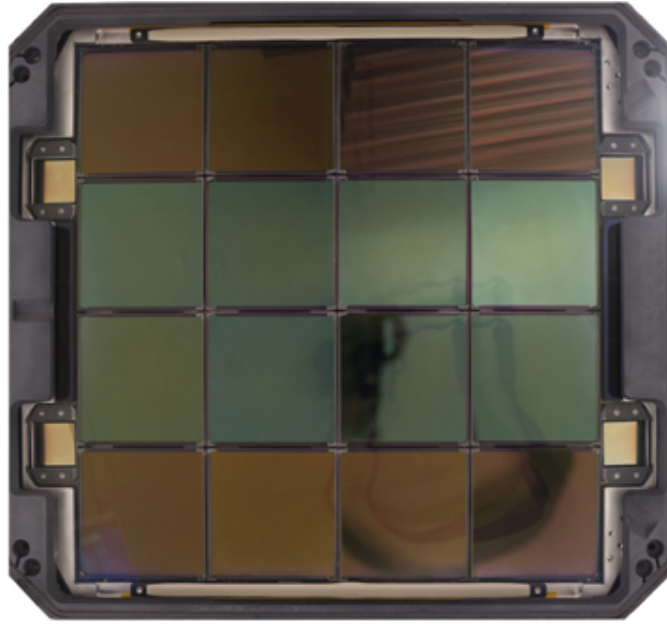


Figure 4.3 – The ZTF camera and its 16 CCDs. Note that the central rows of CCDs are not identical to those at the top and bottom of the camera. The top and bottom rows are optimized for the g and r bands, while the two central rows allow for better observation in the i band (Dekany et al. 2020). Credit: Perley et al. 2020

The ZTF survey stands out from older surveys in particular through its wide field of view. Indeed, its optical system combined with its large camera makes it possible to cover a solid angle of $\sim 47 \text{ deg}^2$ on the celestial sphere per exposure. In Fig. 4.4 the field of view of ZTF is compared to that of other surveys.

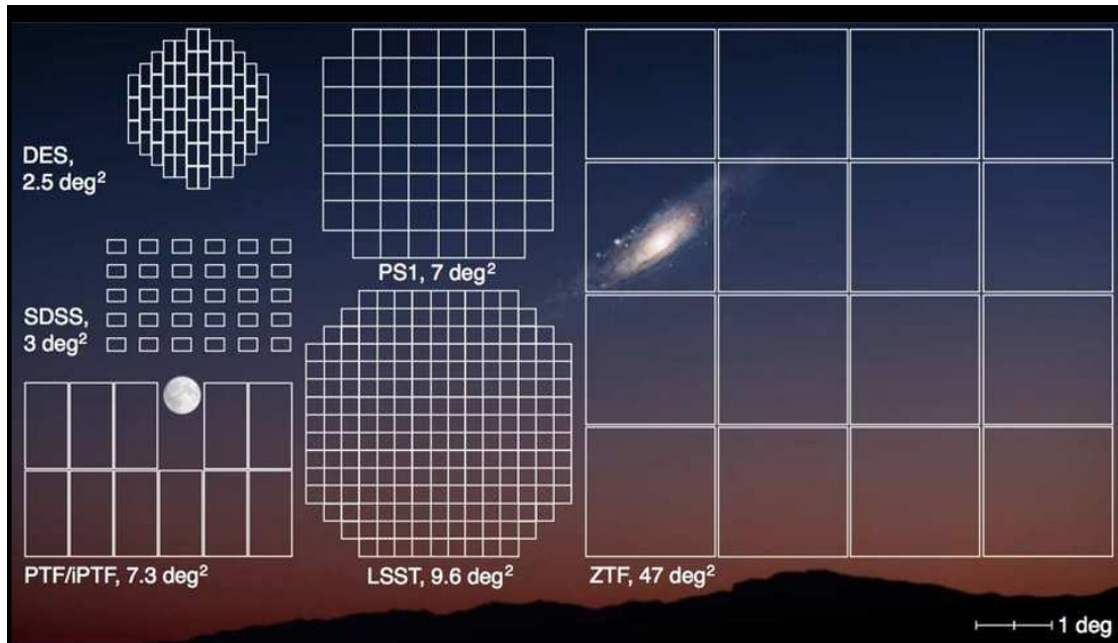


Figure 4.4 – Comparison of the ZTF field of view with those of other surveys. The Moon and the Andromeda galaxy are shown for scale. Credit: Joel Johansson.

ZTF photometry is carried out through three filters g , r and i , which are exchanged in ~ 110 seconds by a robotic arm. Fig. 4.5 shows the transmittance of the three ZTF filters, taking into account the quantum efficiency of the CCDs. In Fig. 4.5, the effect of the atmosphere is not included, in particular the absorption lines are not present in the filter transmittance; however, this is a parameter whose intensity varies with weather conditions as well as with the thickness of atmosphere along the line of sight.

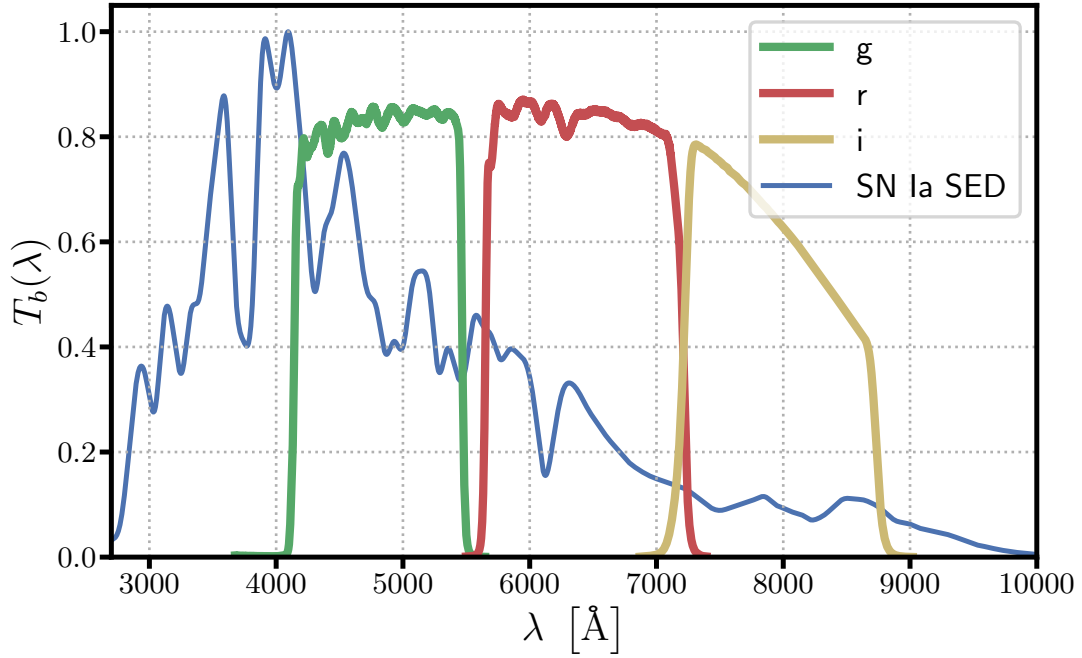


Figure 4.5 – Transmission of the 3 ZTF filters, including the quantum efficiency of the CCDs. The spectrum of a standard SN Ia (SALT2 model) at peak luminosity and at *redshift* $z = 0$ is shown in blue.

The choice of exposure time for a photometric survey is a trade-off between observing more deeply and covering more area in the same amount of time; for ZTF the exposure time is fixed at 30 seconds. The depth of the survey is measured using the 5- σ limiting magnitudes. These magnitudes correspond to those of point sources for which the received signal is five times greater than the noise. The limiting magnitudes in the three ZTF bands are $m_g \sim 20.8 - 21.1$, $m_r \sim 20.6 - 20.9$ and $m_i \sim 19.9 - 20.2$; the variations are largely due to the phases of the Moon. They are obtained for each observation, and their calculation is detailed in Masci et al. 2018.

4.2.2 The ZTF cadence

When observing transient astrophysical objects, and in particular for SNe Ia, it is important to be able to observe the same region of the sky multiple times. The rate at which a telescope observes is called the cadence. In the case of the ZTF survey, the cadence is tailored to the observation of transient phenomena, such that the entire sky of the northern hemisphere is imaged in the g and r bands in ~ 3 days. The cadence of the i band is lower. The set of ZTF fields forms a regular grid. Fig. 4.6 shows the average number of visits per month for each ZTF field in each of the three filters.

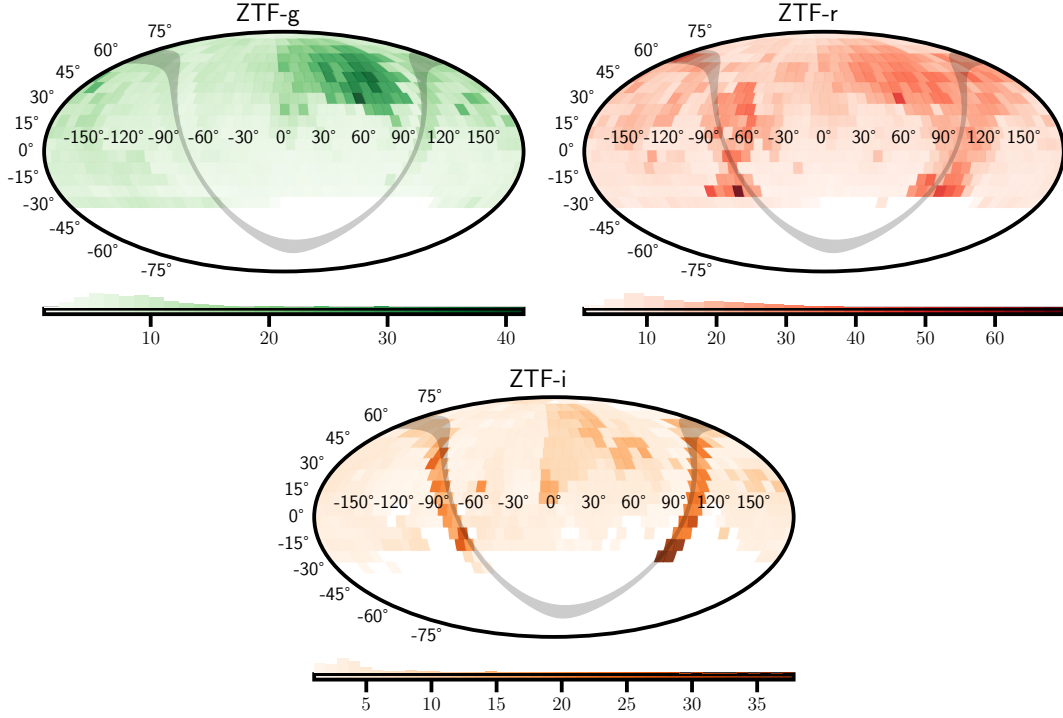


Figure 4.6 – Average number of visits per month in each ZTF field for the three filters g, r and i. The observations used are those between March 2018 and December 2020. The gray band represents the Galactic plane of the Milky Way. Figure produced using the Python library `ztf-fields`.

4.2.3 Spectroscopic follow-up

One of the advantages of the ZTF survey is that it has a spectroscopic follow-up program that allows for the classification of transient phenomena. The spectroscopic survey mainly used for ZTF follow-up is the *Bright Transient Survey* (BTS) (Fremming et al. 2020; Perley et al. 2020), which uses the SED machine (SEDm), a spectroscopic instrument also located at the Palomar Observatory. The BTS survey allows for the classification of almost all transient phenomena observed by ZTF with a magnitude below 19 (Perley et al. 2020).

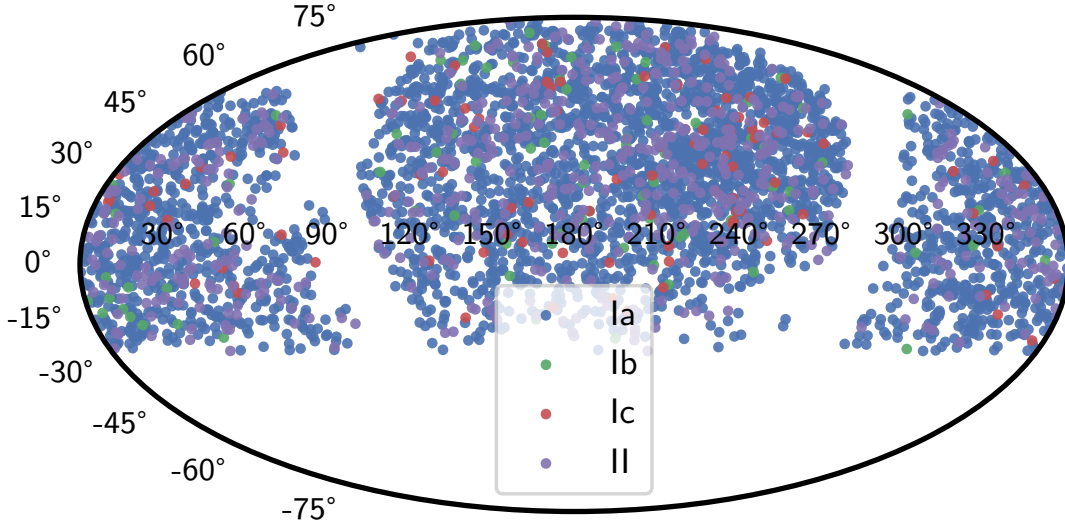


Figure 4.7 – SNe classified by the BTS survey. The most numerous are the SNe Ia, shown as blue points, SNe Ib are in green, SNe Ic in red, and SNe II in purple.

Fig. 4.7 shows the spatial distribution of the supernovae classified by BTS.

4.3 Cosmology and synergies with other surveys

The study of cosmology within the ZTF collaboration therefore focuses on SNe Ia. Indeed, the SNe Ia observed by ZTF will constitute the largest dataset of SNe Ia at low *redshift* available to date. A first data release (DR1) (Dhawan et al. 2021) focuses on presenting the data of 761 spectroscopically classified SNe Ia and on comparing their light-curve parameters with those from earlier surveys. A new data release (DR2) is expected during 2023 (Rigault et al. in prep) with this time ~ 3600 SNe Ia. These statistics will make it possible, in particular, to carry out numerous studies on the various properties and populations of SNe Ia (Dimitriadis et al. in prep, Burgaz et al. in prep, Johansson et al. in prep), on correlations with their environment (Ruppin et al. in prep, Burgaz et al. in prep), as well as on their standardization (Ginolin et al. in prep, Dhawan et al. in prep). Using the ZTF DR2 data to measure luminosity distances precise enough for cosmological analyses will only be possible after a precision photometric calibration effort, currently in preparation, which should be published afterward. Moreover, the ZTF SNe Ia alone will not make it possible to obtain strong cosmological constraints. It is only with the contribution of other surveys that the ZTF SNe Ia can be properly exploited. We give a few examples of surveys with which synergies with ZTF are possible.

4.3.1 Redshifts and the growth rate of structure: ZTF-DESI

In Chap. 3, we explained that cosmology with SNe Ia requires measuring their *redshifts*. The ZTF spectroscopic follow-up, the SEDm, allows some *redshifts* to be measured either from the spectrum of the SN Ia's host or directly from the spectrum of the SN Ia. The precision of these *redshifts* remains rather low, with respective errors of $\sigma_z \sim 10^{-3}$ and $\sigma_z \sim 2 \times 10^{-3}$, i.e. $\sim 10\%$ in relative terms. For part of the ZTF SNe Ia, the *redshifts* will be obtained by searching for the host galaxy of the SN Ia in the catalogs of previous spectroscopic surveys. The *redshifts* thus recovered have generally been measured with good precision, $\sigma_z \sim 10^{-5}$. Improving the precision of *redshift* measurements reduces the errors on most cosmological parameters. This precision is all the more important for measuring the growth rate of structure. Indeed, we saw in Chap. 3 that the information on peculiar velocities is mostly contained in the Doppler shift of the *redshift*. This shift is of order $\Delta z \sim 10^{-3}$. It therefore becomes difficult to extract this information if the error is larger than this shift. A survey that would make it possible to obtain the *redshifts* of a large fraction of the host galaxies of ZTF SNe Ia is the DESI survey. Fig. 4.8 shows the overlap between the sky regions observed by ZTF and DESI. The region covered by DESI is obtained using the Python library `desimodel`¹. We see that the DESI survey is entirely contained within the area observed by ZTF, and will therefore be able to provide part of the *redshifts* needed for ZTF. To obtain the full set of *redshifts*, additional observations will be needed, either through an extension of DESI or through another survey.

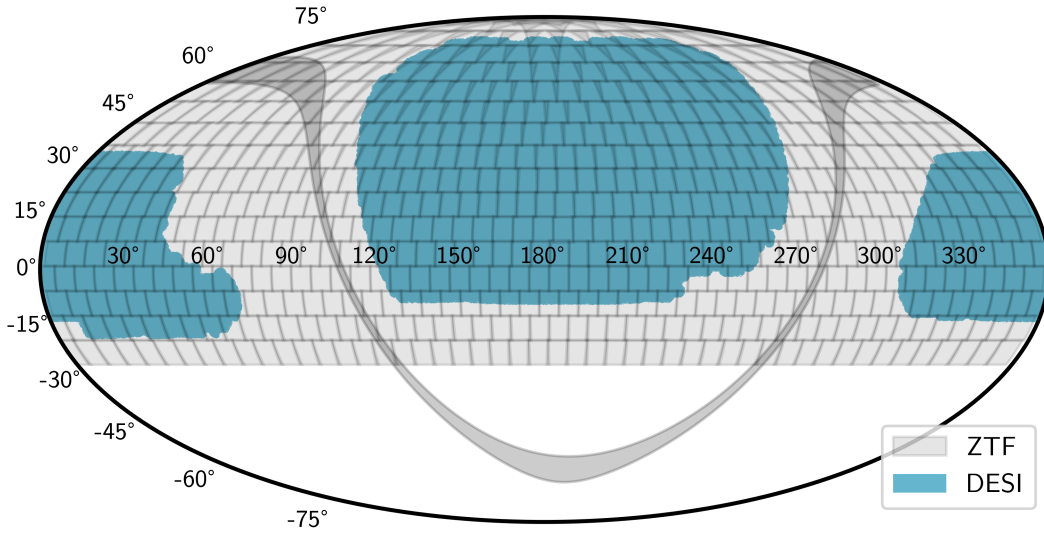


Figure 4.8 – Comparison of the fields of the ZTF and DESI surveys. The gray rectangles represent the ZTF fields, the blue area represents the DESI survey, and the gray band represents the plane of the Milky Way.

1. <https://github.com/desihub/desimodel>

4.3.2 The Hubble constant: ZTF and HST/JWST

The ZTF survey will allow for a measurement of the Hubble constant using a set of SNe Ia measured with the same instrument, which makes it possible to limit certain systematic calibration errors and to have a good understanding of selection biases. However, as discussed in Sec. 1.3.1, measuring the Hubble constant H_0 requires the observation of calibrators. Red giant branch stars measured by HST in some of the ZTF SNe Ia' host galaxies have already been used, and future stars observed by HST and JWST could also be used for this measurement. More details are given in Dhawan et al. 2022.

4.3.3 Constraining dark energy parameters: ZTF and DES

Constraining the dark energy parameters, in particular its equation-of-state parameter w , is an important question in modern cosmology. This measurement requires data over a wide *redshift* range in order to be able to detect any evolution of the dark energy density over the history of our Universe. The ZTF survey only focuses on low *redshift*. The distances measured using ZTF SNe Ia will therefore not be sufficient for this type of measurement. A calibration of the ZTF SNe Ia with SNe Ia at higher *redshift* will be necessary. One example of a possible combination is with the DES survey. Fig. 4.9 shows the complementarity between the ZTF SNe Ia at low *redshift* and those of DES at higher *redshift*.

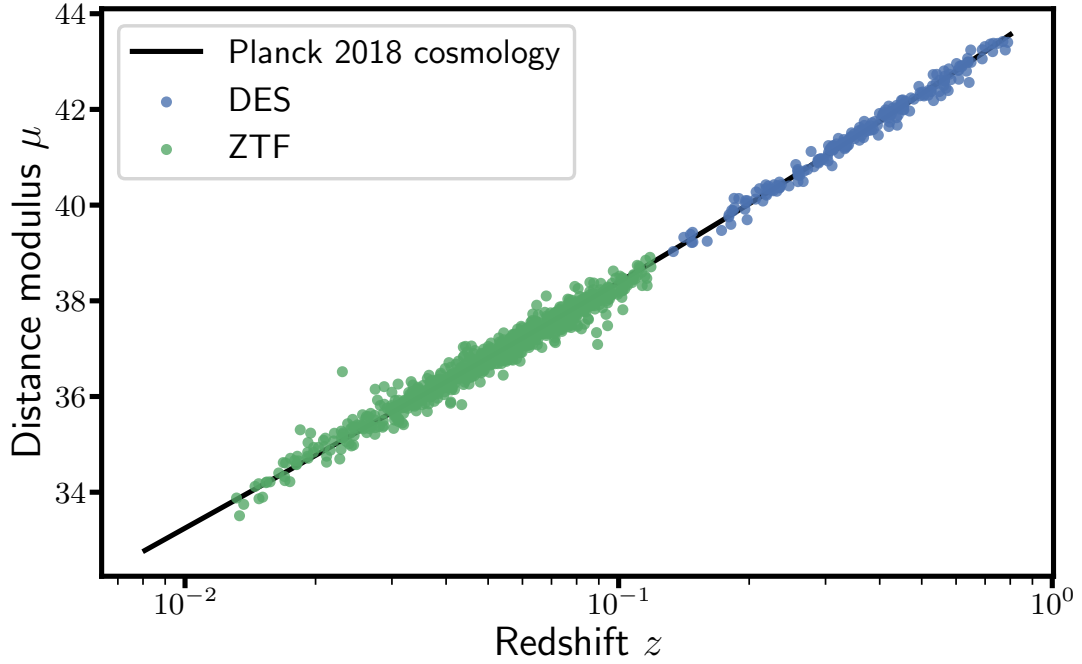


Figure 4.9 – Preview of a ZTF + DES Hubble diagram. The SNe Ia come from the ZTF DR2 and from the DES data (Brout et al. 2019). The cosmology is fiducial and fixed to the values from Planck Collaboration et al. 2020.

In the coming years, a combined analysis with the future SNe Ia measured by the Vera Rubin Observatory should also be carried out. The Vera Rubin Observatory is expected to provide ~ 140000 SNe Ia with a limiting *redshift* of $z_{\text{lim}} \sim 0.3$ for the "Wide-Fast-Deep" (WFD) survey, and between ~ 2000 and ~ 5000 SNe Ia with a limiting *redshift* of $z_{\text{lim}} \sim 0.6$ for the deep-drilling-fields survey (DDF) (Lochner et al. 2022; Gris et al. 2023). These numbers are still subject to change depending on the cadence that will ultimately be adopted by the Vera Rubin Observatory.

5 Simulation and study of selection bias

In modern cosmology, simulations play a crucial role in the interpretation and understanding of our measurements. Advances in computing and the use of large computing centers now make it possible to simulate true "universe boxes," called *N-body* simulations. *N-body* simulations are becoming increasingly precise and make it possible to obtain a realistic dark matter distribution. They are particularly used in preparing analyses of the large-scale structures traced by galaxy surveys. In the context of cosmological analyses with SNe Ia, simulations are generally simpler and consist merely of generating light curves that reproduce the cadence and properties of the surveys to be simulated. In our work on constraining $f\sigma_8$ it is necessary to connect simulations of a SNe Ia survey with the structures of the universe. In this chapter we present how this work was carried out. In Sec. 5.1 we present the data needed to simulate a SNe Ia survey, in Sec. 5.2, we show how we embed our survey simulation within an *N-body* box, Sec. 5.3 describes in more detail the steps of the simulation. In Sec. 5.4 we show how we add the selection function of the ZTF survey. Sec. 5.5 details a comparison between our simulation and the current ZTF data. Finally Sec. 5.6 describes the analysis of our simulation in the context of estimating the peculiar velocities of SNe Ia, a crucial step before the determination of $f\sigma_8$.

5.1 Simulation of a SNe Ia survey

The simulation of SNe Ia surveys is an essential tool. This type of simulation makes it possible to identify and quantify the biases and systematic errors present in an analysis, and, if possible, to correct them. Moreover, they make it possible to estimate the errors expected on the cosmological parameters and to understand, ahead of an analysis or a survey, which strategy allows them to be reduced.

One of the tools used for the simulation of SNe Ia in the DES and LSST-DESC collaborations is the program SNANA¹ (Kessler et al. 2009). However, its use goes beyond simulation alone, and at the time of this work it did not allow the simulation of SNe Ia within an *N-body* simulation. Our choice was therefore to develop our own simulation code. The Python library SNSim, available on github² and documented³, was developed with the aim of studying the constraint on the structure growth-rate

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1. <https://github.com/RickKessler/SNANA>
 2. <https://github.com/bastien carreres/snsim>
 3. <https://snsim.readthedocs.io/en/main/?badge=main>

parameter with ZTF SNe Ia. However, it is not specific to this analysis and its use can be extended to other cosmological analyses and other surveys. SNSim is largely based on the Python library SNCosmo⁴ (Barbary et al. 2023).



Figure 5.1 – Logo of the Python library SNSim

To obtain realistic light curves, we use data from past ZTF observations. The velocity field is implemented using a catalog derived from an N -body simulation, which we present in Sec. 5.2.

5.1.1 Fiducial cosmology

The choice of a cosmology largely determines the distances at which the SNe Ia are located, allowing us to infer the observed flux. The cosmology is implemented using the cosmology module of the Python library astropy (The Astropy Collaboration et al. 2022). The cosmological parameters of interest are those that come into play in the calculation of the luminosity distance. These are H_0 and the density parameters Ω_i . For consistency, the values of these parameters are the same as those of the N -body simulation used.

5.1.2 Spectral model and global parameters of SNe Ia

For the simulation of light curves we choose to use the spectra of the SALT2 model (see Sec. 3.3.1), which depend on the three variables x_0 , x_1 and c and on time. Wavelength-dependent effects can be added to this spectral model. In this work, we added the extinction effect due to Milky Way dust using the CCM89 model (Cardelli et al. 1989) implemented in SNCosmo. The reddening $E(B-V)$ is computed for each SN Ia using the dust map of Schlegel et al. 1998, recalibrated in Schlafly and Finkbeiner 2011, and implemented in the Python library SFDmap⁵. Although implemented in SNSim, in this work we did not use wavelength-dependent intrinsic scatter models (for example G10 (Guy et al. 2010), C11 (Chotard et al. 2011) or BS22 (Brout and Scolnic 2021)). This choice allows for a simpler first approach and should be investigated in future work.

To assign an absolute magnitude to each SN Ia, we use the Tripp relation (Eq. 3.23), which requires choosing the parameters α , β and M_B . For M_B we must change its value

4. <https://github.com/sncosmo/sncosmo>

5. <https://github.com/kbarbary/sfdmap>

to match the reduced Hubble parameter $h_{\text{fid}} = H_{0,\text{fid}}/100$ of our fiducial cosmology. If $M_{B,\text{mes}}$ is measured with a given value of h we obtain

$$M_B = M_{B,\text{mes}} + 5 \log_{10} \left(\frac{h_{\text{fid}}}{h} \right). \quad (5.1)$$

In this work we use the values obtained in Betoule et al. 2014: $\alpha = 0.14$, $\beta = 3.1$ and $M_{B,\text{mes}} = -19.05$ for $h = 0.7$.

5.1.3 Observational data

To obtain light curves similar to those of the ZTF survey, we use information from real observations already carried out. The starting parameters and data are:

- The observation dates
- The size and pointing directions of the observed fields
- The filters used
- The 5- σ limiting magnitude denoted $m_{5\sigma}$. It corresponds to the magnitude for which the signal-to-noise ratio equals 5.
- The CCD gain in electrons / ADU. This is the conversion coefficient between the number of photo-electrons and the signal produced by the camera.
- The zero point of the observations denoted ZP . It corresponds to the magnitude of a source whose received flux corresponds to 1 ADU.

For ZTF, these observational data are obtained using the Python library `ztfquery`⁶ (Rigault 2018).

In addition to the observational data, the simulation must reproduce the field of view of the survey. For ZTF, we use the dimensions given in Table 3 of Dekany et al. 2020 to reconstruct the field of view down to the level of the quadrants. Fig. 5.2 shows the map of the quadrants modeled in SNsim. It contains the 64 quadrants distributed over 16 CCDs separated by small gaps. In this map, the elements of the ZTF field are approximated by rectangles identified by the four points of their corners. This map is only valid in the reference frame of each pointing and must be projected onto the celestial sphere to obtain the different fields in the reference frame of the survey.

6. <https://github.com/MickaelRigault/ztfquery>

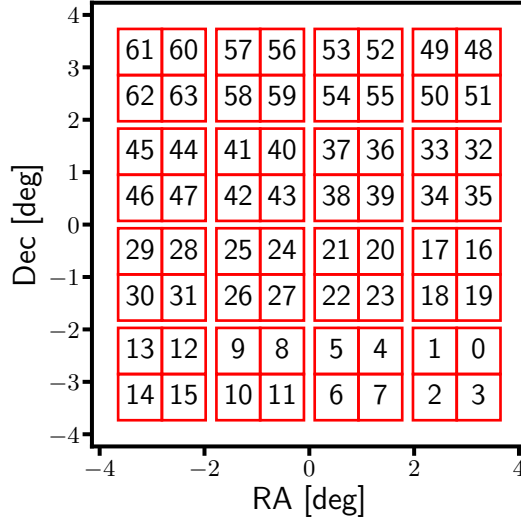


Figure 5.2 – Map of the 64 ZTF quadrants as modeled in SNsim. The quadrants are grouped by 4 on each CCD; note that there are empty gaps between the different CCDs.

The field corresponding to each pointing is obtained by a change of basis that depends on the coordinates (RA, Dec) of each pointing. We denote $\theta_f \equiv \text{RA}$ and $\phi_f \equiv \text{Dec}$ the angular coordinates of a pointing. The Cartesian basis vectors of this pointing are given by

$$\begin{cases} \mathbf{e}'_x = \cos\theta_f \cos\phi_f \mathbf{e}_x + \sin\theta_f \cos\phi_f \mathbf{e}_y + \sin\phi_f \mathbf{e}_z \\ \mathbf{e}'_y = -\sin\theta_f \mathbf{e}_x + \cos\theta_f \mathbf{e}_y + 0 \mathbf{e}_z \\ \mathbf{e}'_z = -\cos\theta_f \sin\phi_f \mathbf{e}_x - \sin\theta_f \sin\phi_f \mathbf{e}_y + \cos\phi_f \mathbf{e}_z \end{cases} \quad (5.2)$$

This decomposition allows us to write the change-of-basis matrix between the reference frame of a pointing $\mathcal{B}'$ and that of the survey $\mathcal{B}$

$$R_{\mathcal{B} \leftarrow \mathcal{B}'} = \begin{pmatrix} \cos\theta_f \cos\phi_f & -\sin\theta_f & -\cos\theta_f \sin\phi_f \\ \sin\theta_f \cos\phi_f & \cos\theta_f & -\sin\theta_f \sin\phi_f \\ -\cos\theta_f \sin\phi_f & 0 & \cos\phi_f \end{pmatrix}. \quad (5.3)$$

The Cartesian coordinates of the quadrant corners in the reference frame of the pointing are given by

$$\mathbf{x}' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta' \cos\phi' \\ \sin\theta' \cos\phi' \\ \sin\phi' \end{pmatrix} \quad (5.4)$$

In practice, we obtain the coordinates of the ZTF field of view for each pointing by applying the transformation

$$\mathbf{x} = R_{\mathcal{B} \leftarrow \mathcal{B}'} \mathbf{x}', \quad (5.5)$$

for the four corners of each of the 64 quadrants of the field of view. The angular

coordinates in the reference frame of the survey are given by

$$\begin{cases} \theta &= \arctan 2(y, x) \\ \phi &= \arcsin\left(\frac{z}{|\mathbf{x}|}\right) \end{cases} \quad (5.6)$$

Once all the transformed point coordinates are obtained, they are connected to form polygons, which constitute our field. Each field obtained in this way is used to select the pointings in which each of the simulated SNe Ia is observed.

5.2 Use of an N -body catalog

In the simulation we need, the velocity field must correspond to a "cosmological" velocity field. That is, the velocities at different positions are correlated with each other in agreement with the power spectrum $P_{\nu\nu}$ corresponding to the simulated cosmology. To achieve this, we use as input to our simulator a catalog derived from an N -body simulation.

5.2.1 N -body simulations

N -body simulations are today an essential tool for cosmology. They are "universe boxes" in which particles are generated and evolved according to the chosen cosmological model and laws of gravity. The particle positions are generated at a sufficiently high *redshift* that the physics there is linear. The particles then evolve to different *redshifts*. Snapshots of these boxes at different *redshifts* provide simulations of a universe at different epochs. Snapshots at different *redshifts* can also be combined to obtain a so-called "light-cone" simulation. In this type of simulation, the distributions of the positions and velocities of the particles evolve with *redshift* in agreement with the cosmological model used. The particles in a box can be grouped into halos, which correspond to matter structures. Such catalogs are then used to prepare future cosmological analyses.

There are many N -body simulations; the most common ones provide a distribution of dark matter particles, generally grouped into halos, such as ABACUSUMMIT (Garrison et al. 2018) or OUTERIM (Heitmann et al. 2019). Other simulations, generally of smaller size, add baryonic physics using hydrodynamical simulation, as is the case for ILLUSTRISTNG (Springel et al. 2018) for example. Finally, some simulations use more advanced methods to simulate realistic effects of the presence of matter on our observations (gravitational lensing, peculiar velocities); examples are the RAYGAL (Rasera et al. 2022) and MICE (Fosalba et al. 2015) simulations.

5.2.2 The OuterRim simulation

During the work carried out for this thesis, the choice of an N -body simulation turned toward the OuterRim simulation (Heitmann et al. 2019). This simulation is

freely available and has been used in many recent eBOSS works (Gil-Marín et al. 2018; Hou et al. 2018; Zarrouk et al. 2018; Avila et al. 2020; Rossi et al. 2021; Smith et al. 2020) and in the DC2 simulations of the Vera Rubin Observatory (LSST Dark Energy Science Collaboration et al. 2021). It is an N -body simulation in a cubic box of side length $L = 3 \text{ Gpc}.h^{-1}$. The fiducial cosmology used in OuterRim is that of the flat ($k = 0$) Λ CDM model. The cosmological parameters of the simulation are given in Table 5.1.

Table 5.1 – Cosmological parameters of the *OuterRim* simulation. H_0 is given in $\text{km.s}^{-1}.\text{Mpc}^{-1}$

H_0	ω_{cdm}	ω_{b}	n_s	σ_8	f	$f\sigma_8$
71	0.1109	0.02258	0.963	0.800	0.478	0.382

Since the ZTF survey is a low-*redshift* survey, we used the $z = 0$ version of this simulation. This simplification neglects the fact that the matter and velocity distributions vary with *redshift*. Over the *redshift* interval $0 < z < 0.1$, $f\sigma_8$ varies by about 5%. The effects of this variation are ignored here but could be the subject of further analyses using light-cone simulations. We divide the OuterRim box into 27 sub-boxes of side length $L = 1 \text{ Gpc}.h^{-1}$. Each of these allows us to simulate an independent realization of ZTF with a maximum *redshift* $z_{\text{max}} \sim 0.17$.

5.2.3 From the N -body catalog to the SNe Ia host catalog

We want to assign to the simulated SNe Ia the velocities corresponding to the velocity field of the N -body simulation. To do this, we assign to each SN Ia a host corresponding to a halo from the catalog of this simulation. Each SN Ia then inherits the position and velocity of its host. This choice is a simplification; in reality each SN Ia is associated with a host galaxy and not with a halo. The position and peculiar velocity of the host galaxy are then slightly different from those of the halo. It should also be added that the explosion rate and the characteristics of SNe Ia within a galaxy can vary depending on the intrinsic properties of the galaxy. We have decided to leave these points to future investigations.

To construct our survey from the catalog, we choose an observer placed at position $\mathbf{x}_{\text{obs}} = (x_{0,\text{obs}}, x_{1,\text{obs}}, x_{2,\text{obs}})$. The coordinates of each halo with initial position $\mathbf{x}_i = (x_{0,i}, x_{1,i}, x_{2,i})$ become

$$\tilde{\mathbf{x}}_i = \mathbf{x}_i - \mathbf{x}_{\text{obs}} = \begin{pmatrix} x_{0,i} - x_{0,\text{obs}} \\ x_{1,i} - x_{1,\text{obs}} \\ x_{2,i} - x_{2,\text{obs}} \end{pmatrix}. \quad (5.7)$$

In the context of a flat universe, the comoving distances are then

$$r_i = \|\tilde{\mathbf{x}}\| = \sqrt{\sum_{j=0}^2 \tilde{x}_{j,i}^2}, \quad (5.8)$$

and the angular coordinates

$$\theta_i = \arctan2(\tilde{x}_{1,i}, \tilde{x}_{0,i}) \quad (5.9)$$

$$\phi_i = \arcsin\left(\frac{\tilde{x}_{2,i}}{r_i}\right). \quad (5.10)$$

where θ is the equivalent of the right ascension (RA) and ϕ is the equivalent of the declination (Dec).

We obtain the *redshift* by numerically inverting the relation $z_{\text{cos}} - r(z_{\text{cos}})$ in the fiducial cosmology

$$z_{\text{cos},i} = r_{\text{fid}}^{-1}(r_i). \quad (5.11)$$

The line-of-sight velocities are computed from the velocity vector of the halos $\mathbf{v}_i = (v_{0,i}, v_{1,i}, v_{2,i})$ as

$$v_{p,i} = \frac{\mathbf{v}_i \cdot \tilde{\mathbf{x}}_i}{r_i} = \frac{1}{r_i} \sum_{j=0}^2 v_{j,i} \tilde{x}_{j,i}. \quad (5.12)$$

The quantities needed to simulate the SNe Ia survey are $z_{\text{cos},i}$, θ_i , ϕ_i and $v_{p,i}$.

5.3 Steps in simulating a SNe Ia survey with SNSim

In this section, we describe the different steps of the survey simulation.

5.3.1 Number of simulated SNe Ia

The number of simulated SNe Ia is computed from an explosion rate r_v in $\text{N.Mpc}^3.\text{yr}^{-1}$ that is given as input to the code. In general, this rate can depend on *redshift*. In this work we use a constant explosion rate that we set to the value measured by Perley et al. 2020

$$r_{v,\text{mes}} = (2.35 \pm 0.24) \times 10^{-5} \text{ SNe Ia.Mpc}^{-3}.\text{an}^{-1}. \quad (5.13)$$

This measurement of r_v is given for a Hubble parameter H_0 fixed such that $h = 0.7$, so, as for M_B , the rate must be changed according to the fiducial cosmology

$$r_v = r_{v,\text{mes}} \left(\frac{h_{\text{fid}}}{h} \right)^3. \quad (5.14)$$

The rate can be constant or a function of *redshift* z . Moreover, the duration of the survey in our reference frame is not the same at different *redshifts*. We therefore divide our volume of the universe into p shells of thickness δz such that

$$\delta z = \frac{z_{\text{max}} - z_{\text{min}}}{p}, \quad (5.15)$$

with z_{min} and z_{max} the *redshift* limits of the simulation. The explosion rate of SNe Ia per year in the observer's reference frame in a thin shell at a mean *redshift* $\bar{z}_i$ is then

given by

$$R_i = \frac{r_v(\bar{z}_i)}{1 + \bar{z}_i} V_i, \quad (5.16)$$

where i is an integer $i \in \{0, \dots, p-1\}$, the factor $1 + \bar{z}_i$ accounts for the time dilation at the different *redshifts*, with the mean *redshift* $\bar{z}_i$ given by

$$\bar{z}_i = z_{\min} + (i+1) \frac{\delta_z}{2}, \quad (5.17)$$

and the volume V_i of each shell is computed as

$$V_i = \frac{4\pi}{3} (r^3(z_{\min} + (i+1)\delta_z) - r^3(z_{\min} + i\delta_z)). \quad (5.18)$$

The average number of SNe Ia to simulate is then

$$\langle N \rangle = \Delta T \times \frac{\Omega_{\text{sky}}}{4\pi} \sum_i R_i, \quad (5.19)$$

where ΔT is the duration of the survey and Ω_{sky} the solid angle of the sky in which the SNe Ia are to be generated. We can assume that the number of SNe Ia that will be observed follows a Poisson distribution with parameter $\lambda = \langle N \rangle$. Finally, we draw the number of simulated SNe Ia N according to this Poisson distribution.

5.3.2 Assigning positions and velocities using the N -body catalog

Our simulation assigns a host from the N -body catalog to each SN Ia. The SNe Ia then have the same positions (z_{cos} , RA, Dec) and velocities as their hosts. We draw N hosts from the catalog, giving each host a weight w_i depending on its *redshift* z_i

$$w_i = \frac{r_v(z_i)}{1 + z_i} \left(\sum_j \frac{r_v(z_j)}{1 + z_j} \right)^{-1}, \quad (5.20)$$

in order to preserve the evolution of the rate with *redshift*. The observed *redshift* of each SN Ia is computed as

$$z_{\text{obs},i} = (1 + z_{\text{cos},i}) \left(1 + \frac{v_{p,i}}{c} \right) - 1. \quad (5.21)$$

5.3.3 Generation of the *stretch* x_1 and color c parameters

Several works have studied the distribution of the x_1 and c parameters. Here we present the distributions used in Carreres et al. 2023.

The x_1 parameters are generated according to the distribution of Nicolas et al. 2021 (Eq. 2), based on the existence of two populations of SNe Ia, one young and the other

older

$$\mathcal{P}(x_1|z) = \delta(z)\mathcal{N}(\mu_1, \sigma_1) + (1 - \delta(z)) [a\mathcal{N}(\mu_1, \sigma_1) + (1 - a)\mathcal{N}(\mu_2, \sigma_2)], \quad (5.22)$$

where $a = 0.51$, $\mu_1 = 0.37$, $\sigma_1 = 0.61$, $\mu_2 = -1.22$, $\sigma_2 = 0.56$ and $\delta(z)$ is the fraction function of the two populations, such that

$$\delta(z) = (K^{-1}(1 + z)^{-2.8} + 1)^{-1}, \quad (5.23)$$

with $K = 0.87$. All numerical values come from Nicolas et al. 2021.

The distribution of the color parameter c is taken from Scolnic and Kessler 2016. It is an asymmetric normal distribution

$$\mathcal{P}(c) = \sqrt{\frac{2}{\pi(\sigma_{\text{high}} + \sigma_{\text{low}})}} \begin{cases} \exp\left(-\frac{(c-\bar{c})^2}{2\sigma_{\text{low}}^2}\right) & \text{if } c < \bar{c} \\ \exp\left(-\frac{(c-\bar{c})^2}{2\sigma_{\text{high}}^2}\right) & \text{if } c > \bar{c} \end{cases}. \quad (5.24)$$

Table 1 of Scolnic and Kessler 2016 gives results for the parameters $\bar{c}$, σ_{low} and σ_{high} . These results indicate a dependence on redshift.

The distributions of x_1 and c are plotted in Fig. 5.3.

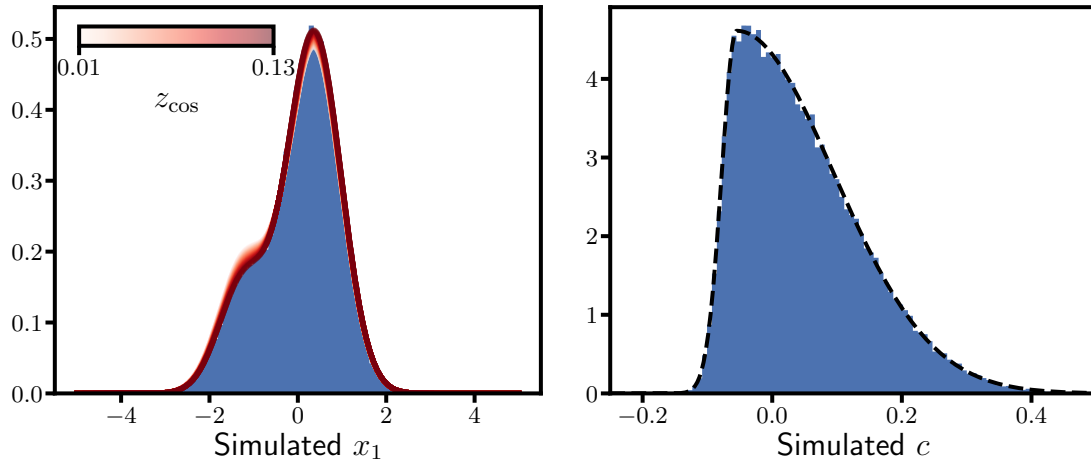


Figure 5.3 – Simulated distributions (in blue) of the *stretch* and color parameters. The left-hand distribution is the distribution of the stretch parameter x_1 from Nicolas et al. 2021; it is a mixture of two Gaussians whose proportion depends on redshift (represented by the variation of the red envelope). It represents the populations of SNe Ia hosted by younger galaxies (dark red envelope) or older galaxies (light red envelope). The right-hand distribution is the distribution of the color parameter c from Scolnic and Kessler 2016; it is an asymmetric Gaussian whose theoretical model is plotted with black dotted lines.

5.3.4 Computation of the magnitude m_B and the parameter x_0

The peak magnitude in the Bessel-B band of the simulated SNe Ia is given by

$$m_{B,i} = \mu_i + M_{B,i}, \quad (5.25)$$

where μ_i is the distance modulus of a SN Ia and is computed as

$$\mu_i = \mu_{\text{model}}(z_{\text{cos},i}) + 10 \log_{10} \left(1 + \frac{v_{p,i}}{c} \right), \quad (5.26)$$

and $M_{B,i}$ is the absolute magnitude of a SN Ia given by the Tripp relation (Eq.3.34)

$$M_{B,i} = M_B - \alpha x_{1,i} + \beta c_i + \sigma_{\text{int},i}, \quad (5.27)$$

where $\sigma_{\text{int},i}$ is the realization for a SN Ia of the intrinsic scatter of the absolute magnitudes of SNe Ia after standardization. The simplest method for modeling this scatter is a normal distribution

$$\mathcal{P}(\sigma_{\text{int}}) = \mathcal{N}(0, \sigma_M), \quad (5.28)$$

where σ_M is the standard deviation of the scatter. Once the parameter m_B is determined, it is converted into the parameter x_0 by the formula

$$x_{0,i} = 10^{-0.4(m_{B,i} - ZP_{AB})} \left(\int_0^{+\infty} \phi_{x_{1,i},c_i}(0, \lambda) T_B(\lambda) \frac{\lambda}{hc} d\lambda \right)^{-1}, \quad (5.29)$$

where $\phi_{x_{1,i},c_i}$ is the SALT2 spectro-temporal model, $T_B(\lambda)$ is the transmittance of the Bessel-B filter and ZP_{AB} is the zero point of the AB magnitude system such that

$$ZP_{AB} = \int_0^{+\infty} \phi_{AB} T_B(\lambda) \frac{\lambda}{hc} d\lambda. \quad (5.30)$$

5.3.5 "True" flux

After generating the temporal, t_0 , and spatial, (z , RA, Dec), location parameters of each SN Ia, we extract from the observation list those for which the SN Ia is observed. We then generate the fluxes corresponding to these observations. Using the SNCosmo library, we can generate a flux in ADU in a band b such that

$$F_{b,i}^{\text{true}}(p_{\text{obs}}) = x_0(1+z)10^{-0.4(ZP_{AB}-ZP)} \times \int_0^{+\infty} \phi_{x_1,c} \left(\frac{p_{\text{obs}}}{1+z_{\text{obs},i}}, \lambda_s \right) T_{\text{MWD}}(\lambda_{\text{obs}}) T_b(\lambda_{\text{obs}}) \frac{\lambda_s}{hc} d\lambda_s, \quad (5.31)$$

where $T_b(\lambda_{\text{obs}})$ is the transmittance of band b and $T_{\text{MWD}}(\lambda_{\text{obs}})$ is the extinction due to Milky Way dust.

5.3.6 Realistic flux

We need to model the noise of each observation. Since photon counting is a Poisson process, we have a variance

$$\sigma_{F_p}^2 = F_p. \quad (5.32)$$

However, the measured signal is not directly the number of photons but a signal F in ADU such that

$$F = \frac{F_p}{G}, \quad (5.33)$$

with G the gain. The Poisson variance on the flux F is

$$\sigma_F^2 = \frac{F}{G}. \quad (5.34)$$

In addition to Poisson noise, noise due to observing conditions affects the flux measurement, such as sky background noise. This noise is included in the $5\text{-}\sigma$ limiting magnitude and can be obtained from it. If we group this noise under the notation of sky noise σ_{sky} we have

$$\frac{F_{5\sigma}}{\sigma_{F_{5\sigma}}} = \frac{F_{5\sigma}}{\sqrt{\sigma_F^2 + \sigma_{\text{sky}}^2}} = 5 \quad (5.35)$$

In Fig. 5.4 we show the 2D histogram of the ratio between the Poisson noise and the sky noise (respectively σ_F^2 and σ_{sky}^2) as a function of the flux equivalent to $5\text{-}\sigma$ for 4 years of ZTF observation data. We note that for the $5\text{-}\sigma$ flux error, the Poisson noise contributes less than 10%. We can therefore approximate

$$\sigma_{\text{sky}} \simeq \frac{1}{5} F_{5\sigma} = \frac{1}{5} 10^{-0.4(m_{5\sigma} - ZP)}. \quad (5.36)$$

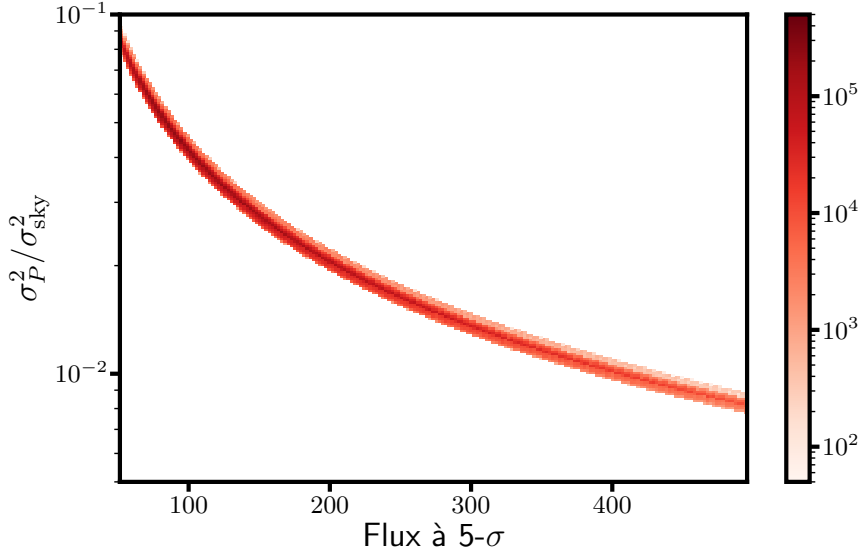


Figure 5.4 – 2D histogram of the ratio between the Poisson variance and the variance due to sky noise as a function of the flux equivalent to 5- σ .

To compare fluxes with each other, the zero-point measurement is necessary. This measurement has an error σ_{ZP} . If we denote the calibrated flux

$$F_{\text{cal}} = F \times 10^{0.4ZP}, \quad (5.37)$$

the error on this calibrated flux is given by

$$\sigma_{F_{\text{cal}}}^2 = (\sigma_F 10^{0.4ZP})^2 + \left(F \frac{\partial}{\partial ZP} 10^{0.4ZP} \right)^2 \quad (5.38)$$

$$= (\sigma_F 10^{0.4ZP})^2 + \left(F \frac{\ln 10}{2.5} \sigma_{ZP} 10^{0.4ZP} \right)^2 \quad (5.39)$$

$$= \left(\sigma_F^2 + \left(F \frac{\ln 10}{2.5} \sigma_{ZP} \right)^2 \right) \times (10^{0.4ZP})^2. \quad (5.40)$$

We can finally include the calibration error term in the flux error

$$\sigma_F^2 = \frac{F}{G} + \sigma_{\text{sky}}^2 + \left(\frac{\ln 10}{2.5} F \right)^2 \sigma_{ZP}^2. \quad (5.41)$$

The output flux of the simulation is a Gaussian realization

$$F \sim \mathcal{N}(F_{\text{true}}, \sigma_F). \quad (5.42)$$

Here we have assumed that all observations are independent. Moreover, in our error model we have neglected certain noise sources such as that due to the host galaxy flux, as well as systematics related to calibration. We do not expect these sources of error

to have a very significant effect on our measurement of $f\sigma_8$, but this will need to be verified in future work.

Fig. 5.5 shows three ZTF light curves simulated with SNSim at three different *redshifts*: the first at a very nearby *redshift* $z = 0.01$, the second at an intermediate *redshift* $z = 0.07$ and the last at a more distant *redshift* $z = 0.12$. We can see the evolution of the errors on the flux measurements.

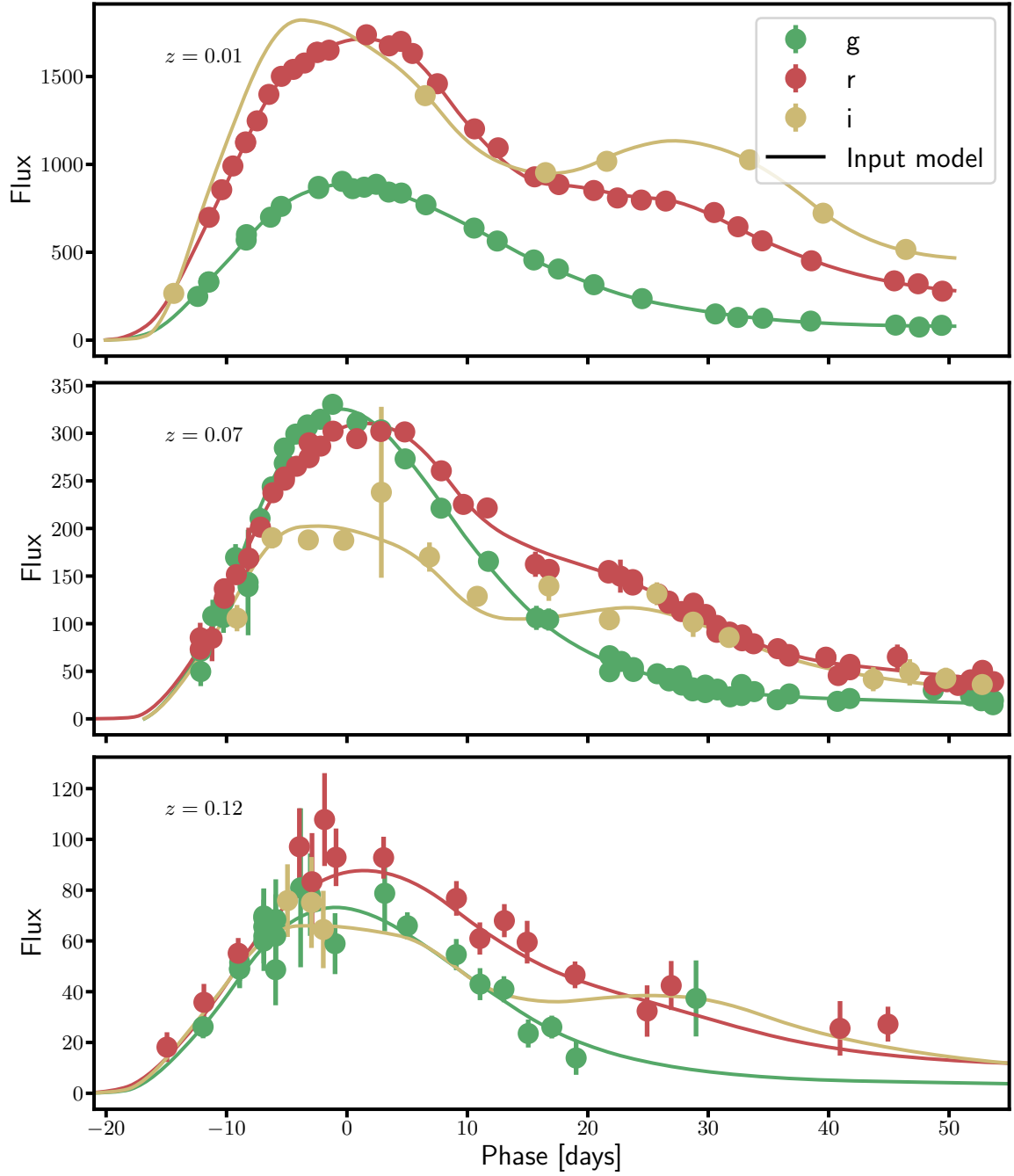


Figure 5.5 – Examples of three light curves at *redshifts* $z = 0.01$, $z = 0.07$ and $z = 0.12$, in a simulation of the ZTF survey with SNSim.

5.4 Adding a selection function

Selection bias (sometimes called Malmquist bias) is described for the first time in (Malmquist 1922; Malmquist 1925). It occurs in magnitude-limited surveys where,

beyond a certain distance, only the brightest objects can be observed. This bias leads to the observation of an incomplete population of SNe Ia.

Selection bias originates from the observational limit of the survey but also from the quality-criterion choices we apply to select the SNe Ia used in our analysis. We will therefore take these effects into account in our simulation.

5.4.1 Detection limit

The farther away a source is, the more its photon flux decreases, until it becomes too weak to be detected. Fig. 5.6 shows the peak magnitude of a SN Ia in the ztf-g band as a function of *redshift*. We have also plotted the median 5- σ magnitude of ZTF in this same band. We can see that the limiting magnitude is reached around $z \sim 0.2$.

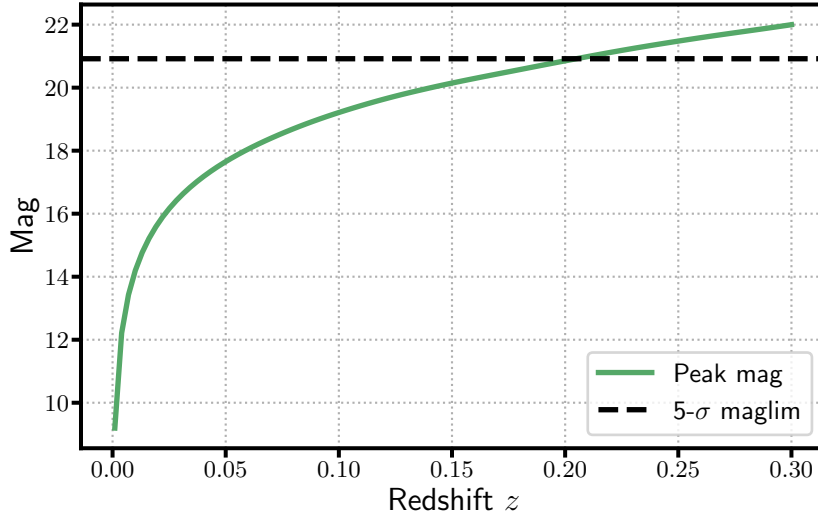


Figure 5.6 – Observed magnitude in the ztf-g band for a SN Ia ($x_1 = c = 0$) as a function of *redshift*. The dashed black line represents the median value of the 5- σ limiting magnitude.

A SN Ia requires several measurements at different dates to be usable in a cosmological analysis; in the simulation carried out in Carreres et al. 2023 we decided to model the detection limit by requiring at least 2 flux measurements with $\text{SNR} > 5$.

5.4.2 Spectroscopic identification selection

Identification of SNe Ia makes it possible to keep only SNe Ia that have the properties of *standard* candles for the analysis.

The ZTF survey benefits from spectroscopic follow-up that allows, by measuring the spectrum of observed objects, their identification. This spectroscopic follow-up is described in Perley et al. 2020 and takes the form of a catalog of identified SNe called the *Bright Transient Survey* (BTS). The identification efficiency is given in

Figure 4 of Perley et al. 2020; it allows us to determine the selection function, that is, the identification probability as a function of magnitude. This function is shown in Figure 5.7; we see that SNe Ia observed by ZTF with a magnitude $m < 17$ are identified with 100% efficiency, this identification efficiency then decreases and reaches $\sim 90\%$ for an observed magnitude of $m \sim 18.5$, and for $18.5 < m < 19$ the efficiency drops from 90% to 50%. Beyond that, for magnitudes $m > 19$ identification is not carried out and the identification probability is therefore zero.

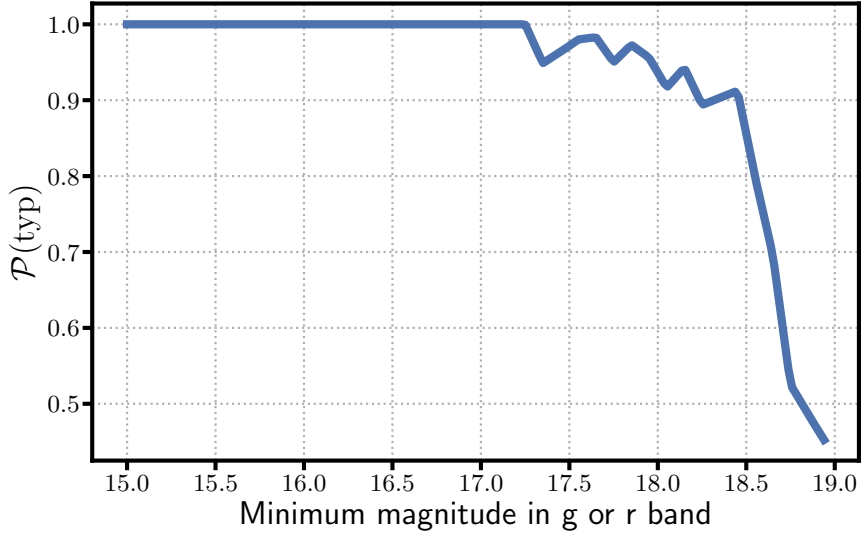


Figure 5.7 – BTS selection function; the identification probability is shown as a function of the minimum magnitude in the ZTF r or g band.

SNe identified in the BTS catalog are subject to additional selection criteria described in section 2.3 of Perley et al. 2020. In Carreres et al. 2023, to simulate the effect of this selection, we decided to apply the first three:

- At least one observation before the observed luminosity peak, such that $-16.5 < t - t_{\text{peak}} < -7.5$ days;
- Around the peak, at least one observation such that $-7.5 < t - t_{\text{peak}} < -2.5$ or $2.5 < t - t_{\text{peak}} < 7.5$;
- After the luminosity peak, at least one observation such that $7.5 < t - t_{\text{peak}} < 16.5$ or two observations, one such that $2.5 < t - t_{\text{peak}} < 7.5$ and one such that $16.5 < t - t_{\text{peak}} < 28.5$

where t_{peak} is the date of the brightest observation with $\text{SNR} > 5$.

To reproduce a SNe Ia catalog comparable to that of the DR2, after the simulation we apply the selection criteria and then perform a random draw according to the probability law of Figure 5.7. For this draw, we use the minimum magnitude in the g or r band. To avoid randomly low magnitudes, we only consider observations with $\text{SNR} > 5$.

5.4.3 Simulated datasets

Using the steps described in the previous sections, we simulated two different ZTF realizations for each of our 27 sub-boxes. The first is a realization in which we increased the supernova rate to match the equivalent of six years of data taking. The second simulation is similar to the ZTF DR2 (Rigault et al. in prep.) which gathers the SNe Ia observed over the first three years of ZTF between March 2018 and December 2020.

5.5 Comparison of the simulation with the ZTF DR2

The simulation work remains imperfect and must be compared with data in order to identify differences and be correctly interpreted. In this section we compare the simulation with the ZTF DR2 data. For this comparison we use the DR2 supernovae that have been identified as type Ia, and we apply identical cuts to make our simulation of the DR2 comparable to the data. We thus choose to select the *redshift* range $0.01 < z < 0.12$ and the time range $58270 \text{ MJD} < t_0 < 59210 \text{ MJD}$.

Our selection is based on the identification of SNe Ia by BTS, but some DR2 SNe Ia are identified by other surveys. We therefore restrict the data to SNe Ia present in BTS that satisfy the cuts used in the simulation. We then have 1948 SNe Ia in the data compared to 1831 on average in our simulation. Part of the difference in the number of SNe Ia may come from the explosion rate used in the simulation.

5.5.1 Redshift distribution

In Fig. 5.8 we show the *redshift* distribution of the simulation (in red) compared to the DR2 SNe Ia identified in BTS (in blue).

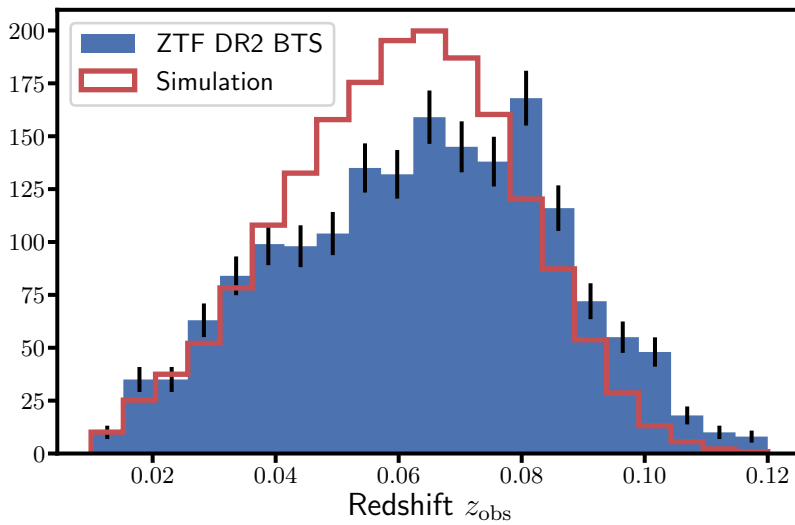


Figure 5.8 – *Redshift* distribution of the ZTF DR2 SNe Ia and of the simulation.

5.5.2 Distribution of SALT2 parameters

In Fig. 5.9 we show, from top to bottom, the parameters x_0 , x_1 and c . The distribution of the x_0 parameter of the simulation appears to match what is found in DR2-BTS. For the *stretch* parameter x_1 , we see the Gaussian mixture distribution proposed in Nicolas et al. 2021 appear in the data, but the low-*redshift* Gaussian seems to have a lower mean value than the one found in Nicolas et al. 2021 and used in our simulation; future analyses will make it possible to update the model. For the values of the color parameter c , we see that the simulation distribution is close to that of the data. The differences, notably at the peak of the distribution, can be explained by the limited statistics of low-*redshift* SNe Ia available to fit the distribution in Scolnic and Kessler 2016.

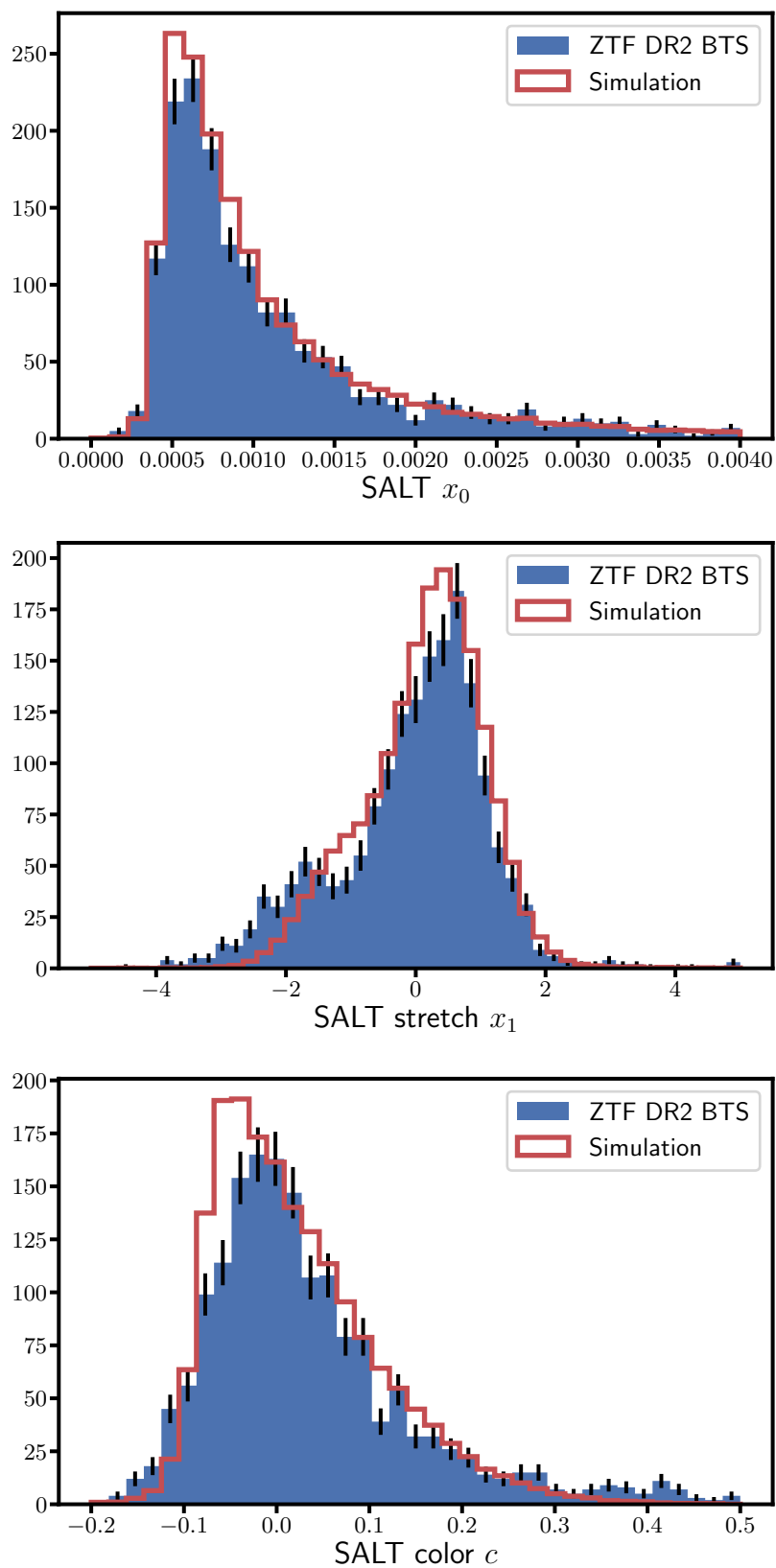


Figure 5.9 – Distribution of the SALT2 parameters of the ZTF DR2 SNe Ia and of the simulation.

5.5.3 Flux, noise and SNR

In Fig. 5.10, we compare the fluxes, errors and SNR of the DR2-BTS and simulation observations. To make the histograms comparable, we apply a weight equal to the ratio of the number of observations in the data and in the simulation. The flux agreement appears to be good. For the errors, the simulation appears to contain an excess of observations with low errors compared to the data.

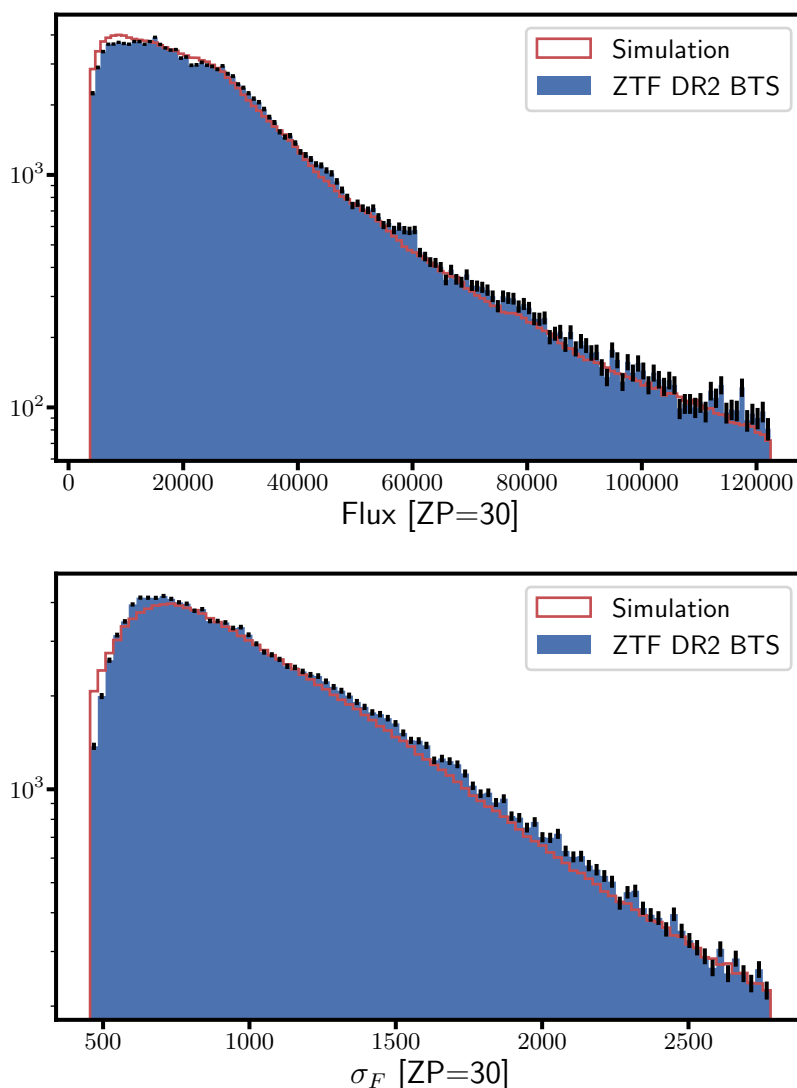


Figure 5.10 – Distribution of the fluxes and errors of the ZTF DR2-BTS observations (in blue) and of the simulations (in red). The limits are set as the 1st and 95th percentiles.

Fig. 5.11 compares the SNR distributions of the ZTF DR2-BTS observations and the simulation. The two distributions are similar for SNR values < 20 but then diverge. We note that the simulation appears to have an excess of observations with SNR between

20 and 40 compared to the data. At high SNR, the simulation distribution plateaus around an SNR value of 100, whereas the data have a broad distribution tail beyond that.

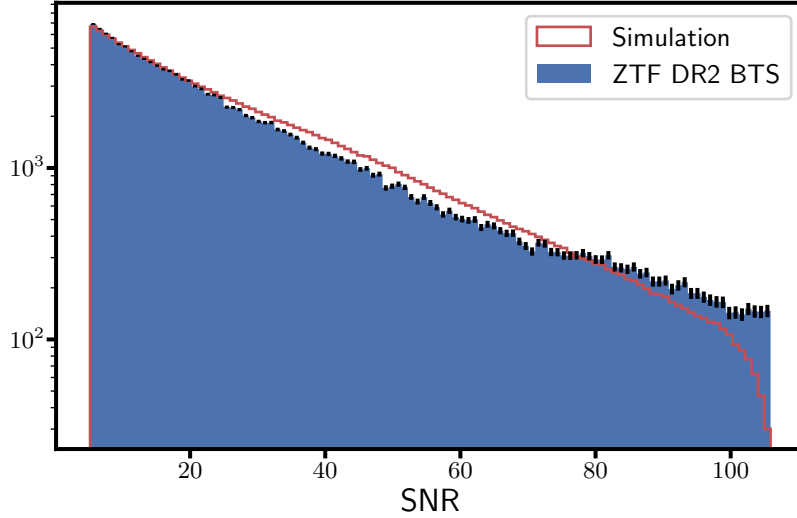


Figure 5.11 – Distribution of the SNR of the ZTF DR2-BTS observations (in blue) and of the simulations (in red). The limits are set as the 1st and 95th percentiles.

Although our simulation provides a good reproduction of the data, it shows a few differences that are not yet fully understood. The development of the new calibration for ZTF photometry (see Sec. 4.3) should allow a better understanding of the errors and a better account of them in the simulation.

5.6 Study of the simulation for velocity estimation

The use of the *OuterRim* N-body simulation with SNSim allowed us to produce 27 independent realizations of ZTF with a *redshift* between 0.01 and 0.17. These simulations will allow us to study the analysis of the measurement of the structure growth rate, for which the preliminary step is velocity estimation.

5.6.1 Selection bias

As discussed in Sec. 5.4, the survey selection functions reproduced in our simulation will result in a selection bias that we want to study here. Fig. 5.12 shows the angular distribution of SNe Ia before and after selection; the SNe Ia before selection are represented by blue points, the SNe Ia that pass the detection criterion are in green, and those that pass the identification criterion are in red. We can see the effect of these selections in terms of the angular position of the SNe Ia. The most heavily filtered SNe Ia are those whose line of sight passes through the galactic plane, where the density of stars and dust reduces the quality of the observations.

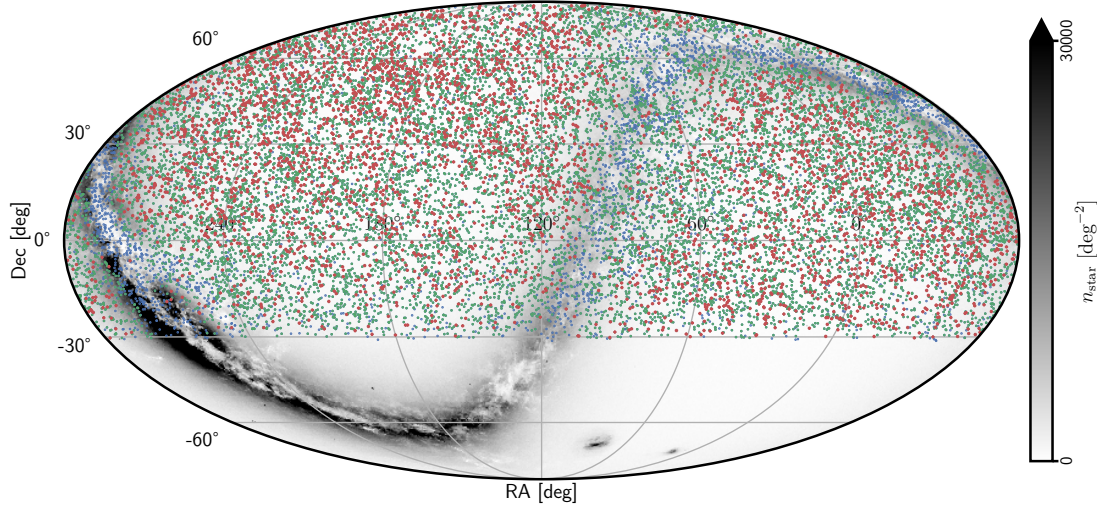


Figure 5.12 – Angular distribution of the simulated SNe Ia in one of the ZTF realizations. The SNe Ia before selection are shown in blue, those that pass the detection criterion are in green and those that pass the classification criteria are in red. The background shows a stellar density map measured by the Gaia satellite.

In Fig. 5.13, we show the effect of the selection in terms of *redshift*. The top part of the figure shows the *redshift* distribution. The bottom part shows the density of SNe Ia as a function of *redshift*. We can see that the distribution of SNe Ia before selection (in blue) decreases as $(1+z)^{-1}$ due to the fact that the duration of the survey in the observer's reference frame is contracted at higher z . The SNe Ia that pass the detection criterion (in green) see their number decrease by about $\sim 40\%$, with a distribution that decreases faster than $(1+z)^{-1}$ as *redshift* increases. The distribution of identified SNe Ia (in red) represents a bit less than $\sim 40\%$ of the distribution before selection and undergoes a rapid decrease as *redshift* increases; almost no SNe Ia remain beyond $z \sim 0.10$.

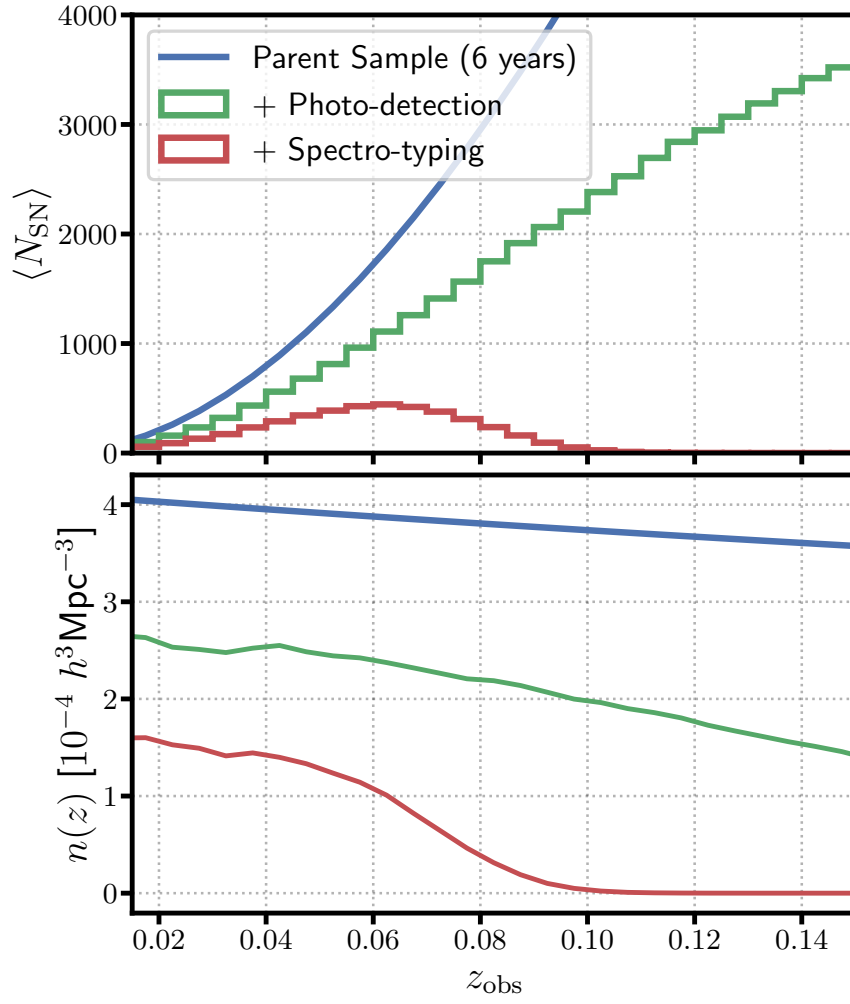


Figure 5.13 – *Top*: Distribution of SNe Ia *redshifts*. *Bottom*: Density of SNe Ia. In blue: SNe Ia before selection. In green: SNe Ia after detection criteria. In red: SNe Ia after spectroscopic identification selection.

Fig. 5.14 shows the Hubble diagram constructed with the simulated values of the SNe Ia parameters. We see that, for the identified SNe Ia (in red), the distribution of the residuals is biased toward negative values. This is the effect of selection bias.

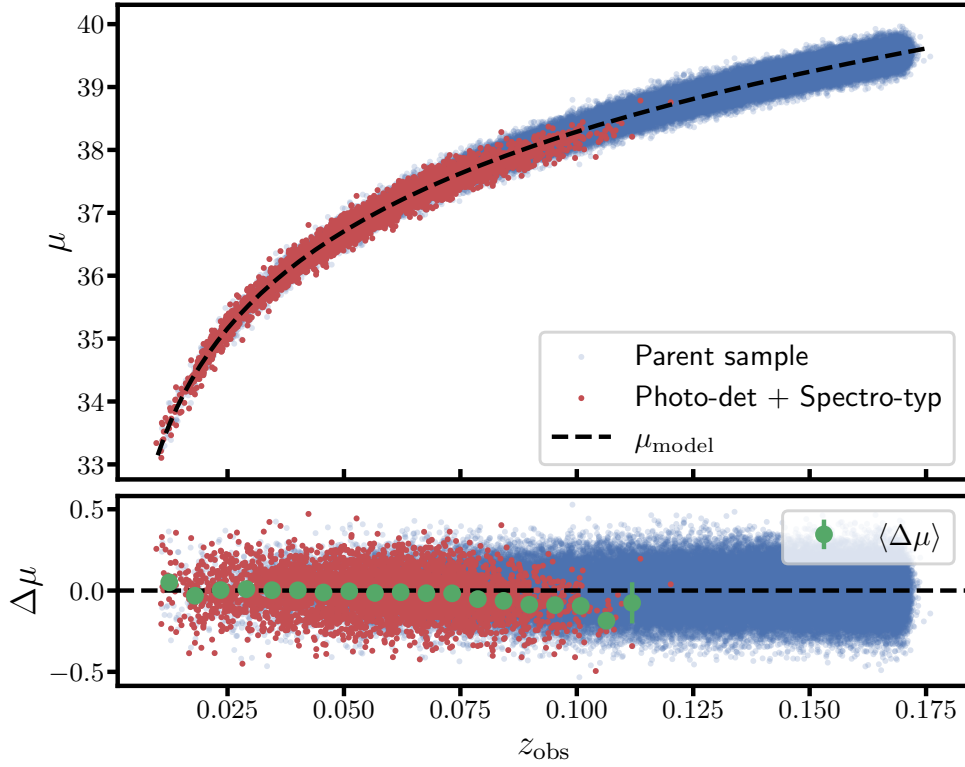


Figure 5.14 – Hubble diagram and residuals of the simulated SNe Ia. All simulated SNe Ia are shown in blue, the SNe Ia that pass the selection are in red.

This selection is purely related to the magnitude of the objects and does not introduce, at this stage, a bias on the true velocity population. This is what we see in Figure 5.15, where we show, as a function of *redshift*, the velocities of the detected and identified SNe Ia (in red), their average per *redshift* bin (in green), and the average velocity in the original OuterRim box (in blue). This observation could turn out to be different for real data. For example, the difficulty of detecting SNe Ia in the most massive host galaxies could induce a selection bias on velocities if these galaxies have a velocity distribution significantly different from that of other galaxies.

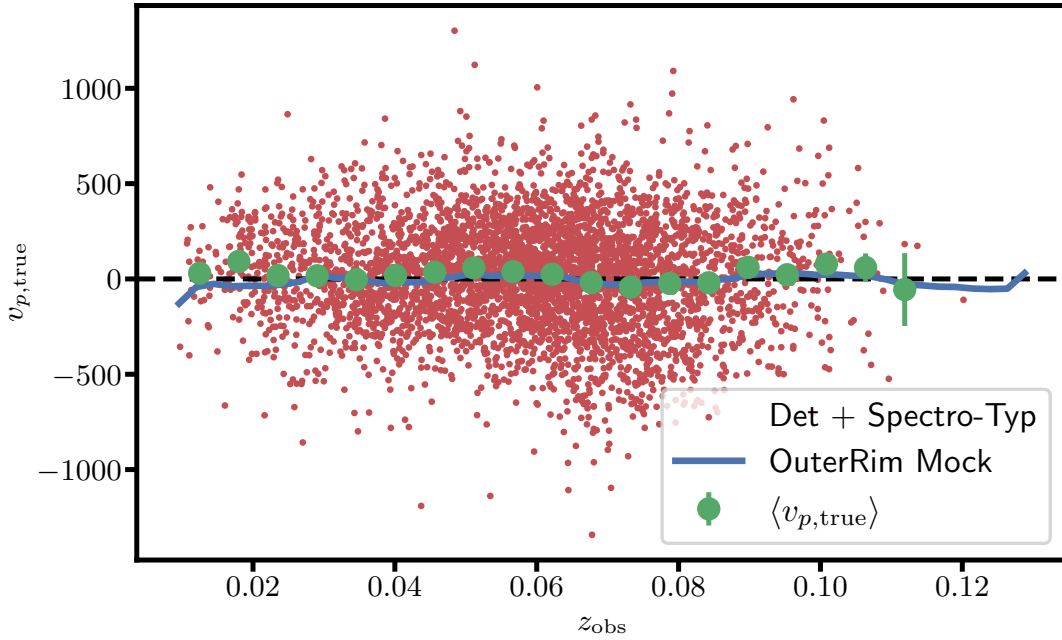


Figure 5.15 – Velocities of the SNe Ia hosts for one of the ZTF realizations as a function of *redshift*. The individual velocities of the hosts are in red, the average per *redshift* bin is in green and the average for the OuterRim box velocities is shown in blue.

5.6.2 Light-curve fitting

The fitting of each light curve is performed using the implementation of the SNeCosmo fitting algorithm. This uses the `Minuit` minimizer from the Python library `iminuit`⁷ (James and Roos 1975).

During the first attempt, the fitting of some SNe Ia struggled to converge. One reason concerns light curves with high SNR, whose fit has an excessively large χ^2 (Fig. 5.16), indicating an underestimation of the errors or convergence of the fit to a false minimum. To address this problem, we perform a two-step fit: in the first step we minimize the χ^2 while artificially increasing the flux errors, which helps avoid falling into false minima. We then perform a second fit with the true errors, but using the parameters found in the first iteration as the starting point. A second reason for poor fit convergence is the large number of observations near the boundaries of the SALT2 domain of validity. Indeed, during χ^2 minimization, the peak-luminosity date parameter t_0 is varied. However, the SALT2 model is defined for $-20 < t_s - t_0 < 50$. During the minimization iterations, some observations move in and out of the SALT2 domain of validity, making the fit unstable. This problem is solved by removing from the fit points such that $t - t_0 > 45$ or $t - t_0 < -15$.

7. <https://github.com/scikit-hep/iminuit>

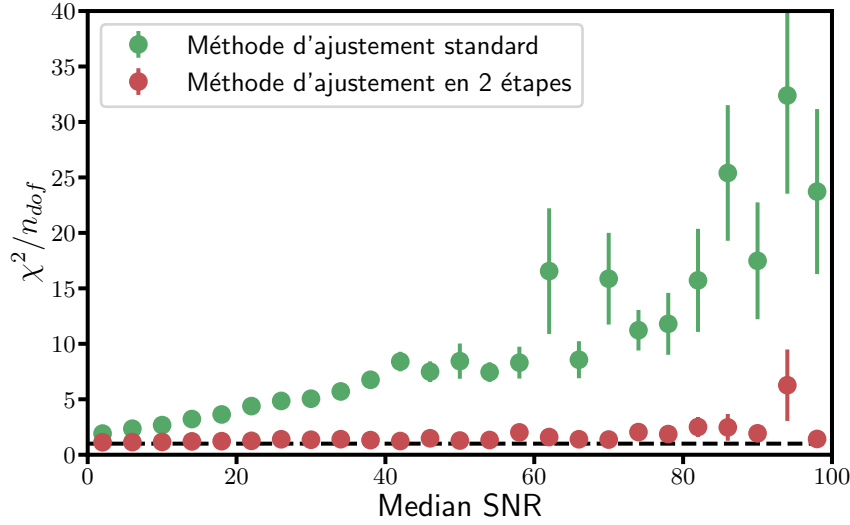


Figure 5.16 – Average reduced χ^2 as a function of the median signal-to-noise ratio of the simulated light curves. Results for the standard fitting method are shown as green points, those for the two-step fitting method are shown as red points

After fitting, we apply quality criteria to the SNe Ia. These criteria are partly taken from Dhawan et al. 2021 and are summarized in Table 5.2.

The first criterion is the good convergence of the χ^2 minimization by Minuit. The second criterion is a model rejection probability with $\alpha = 0.05$. This selection amounts to computing, for each fit, the "cut" value χ^2_{cut} such that

$$\mathcal{P}(\chi^2 < \chi^2_{\text{cut}}) = 1 - \alpha \Leftrightarrow \int_0^{\chi^2_{\text{cut}}} f_{\chi}(x; k) dx = 1 - \alpha, \quad (5.43)$$

where $f_{\chi}(x; k)$ is the probability density of the χ^2 distribution with k degrees of freedom. The fit of the SN Ia is rejected if $\chi^2 > \chi^2_{\text{cut}}$. We note here that when this criterion was applied after the first fitting attempt, only $\sim 63\%$ of the SNe Ia passed this criterion; the second fitting method allowed this to increase to $\sim 85\%$. The following criteria ensure the quality of the SNe Ia fit. We require at least three points within a 10-day window around the luminosity peak. We discard SNe Ia whose x_1 and c parameters are too far from the known distributions, keeping only SNe Ia such that $|x_1| < 3$ and $|c| < 0.3$. To remove fits with excessively large errors, we discard SNe Ia such that $\sigma_{t_0} > 1$ or $\sigma_{x_1} > 1$. Finally, to avoid entering the regime in which biases on peculiar velocities become too large, we impose $z_{\text{obs}} > 0.02$.

5.6.3 Hubble diagram residuals and velocity estimation

The Hubble diagram residuals depend on the parameters $\mathbf{p}_{\text{HD}} = \{M_B, \alpha, \beta, \sigma_M\}$. In Figure 5.17 we show the Hubble diagram residuals averaged over the 27 realizations

Table 5.2 – Selection criteria applied to the simulated ZTF SNe Ia to reproduce a dataset for cosmological analyses.

Selection criteria	SNe Ia remaining %	$\langle N \rangle$ SNe Ia
Valid SALT2 fit	88.7	3830
$P_{\text{fit}} > 95\%$	84.9	3664
3 observations with $ p < 10$	89.7	3873
$ x_1 < 3$	89.5	3867
$ c < 0.3$	88.8	3834
$\sigma_{t_0} < 1$	89.4	3862
$\sigma_{x_1} < 1$	89.3	3858
$z_{\text{obs}} > 0.02$	97.9	4228
All criteria combined	81.5	3520
All criteria combined and $z < 0.06$	38.5	1660

as a function of redshift. The Hubble diagram parameters $\mathbf{p}_{\text{HD}}$ are fixed to the values used as input to the simulation. As in Figure 5.14, we can see that from a *redshift* $z \sim 0.06$ onward, the residuals show a negative bias due to the selection function. This bias reaches a maximum of $\Delta\mu \sim -0.13$ at $z \sim 0.12$.

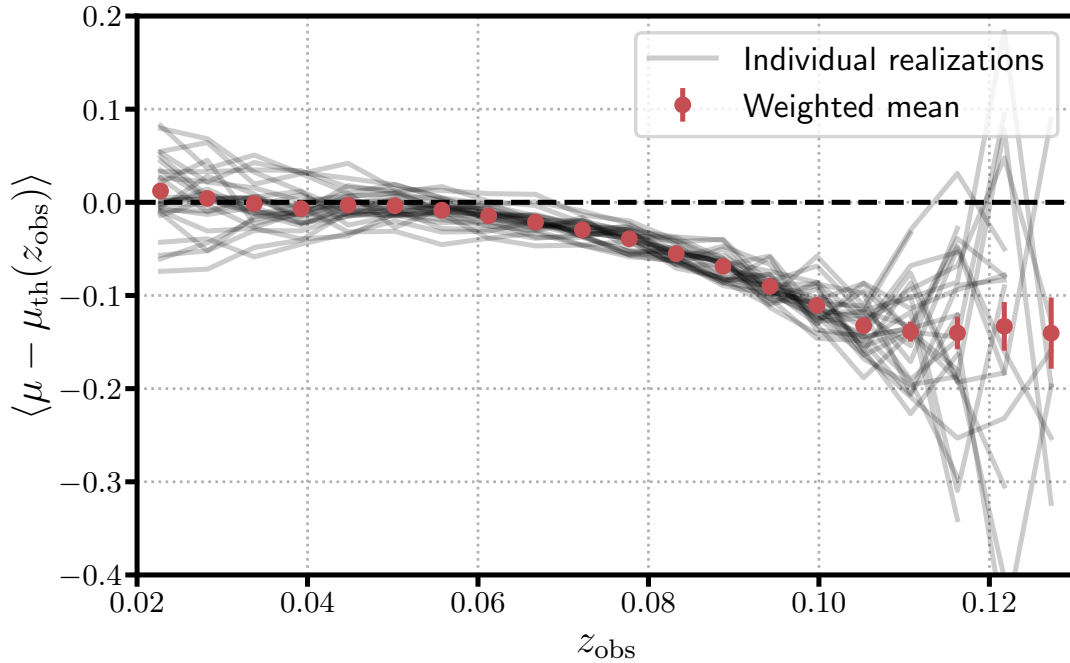


Figure 5.17 – Hubble diagram residuals for the 27 ZTF realizations as a function of *redshift*. The gray lines represent each realization individually, the red points represent the average over all SNe Ia.

We use the velocity estimator presented in (3.60)

$$\hat{v}_i(\mathbf{p}_{\text{HD}}) = -\frac{\ln(10)c}{5} \left(\frac{(1 + z_{\text{obs},i})}{H(z_{\text{obs},i}) r(z_{\text{obs},i})} - 1 \right)^{-1} \Delta\mu_i(\mathbf{p}_{\text{HD}}), \quad (5.44)$$

to obtain the velocities corresponding to the residuals $\Delta\mu(\mathbf{p}_{\text{HD}})$. The difference between these estimated velocities and the true velocities is shown as a function of *redshift* in Figure 5.18. We observe that the bias on the residuals seen in Figure 5.17 is carried over into the velocity estimation. From $z \sim 0.06$ onward, the bias on the velocity estimation increases progressively, reaching $\Delta v \sim 2000 \text{ km.s}^{-1}$ at $z \sim 0.12$.

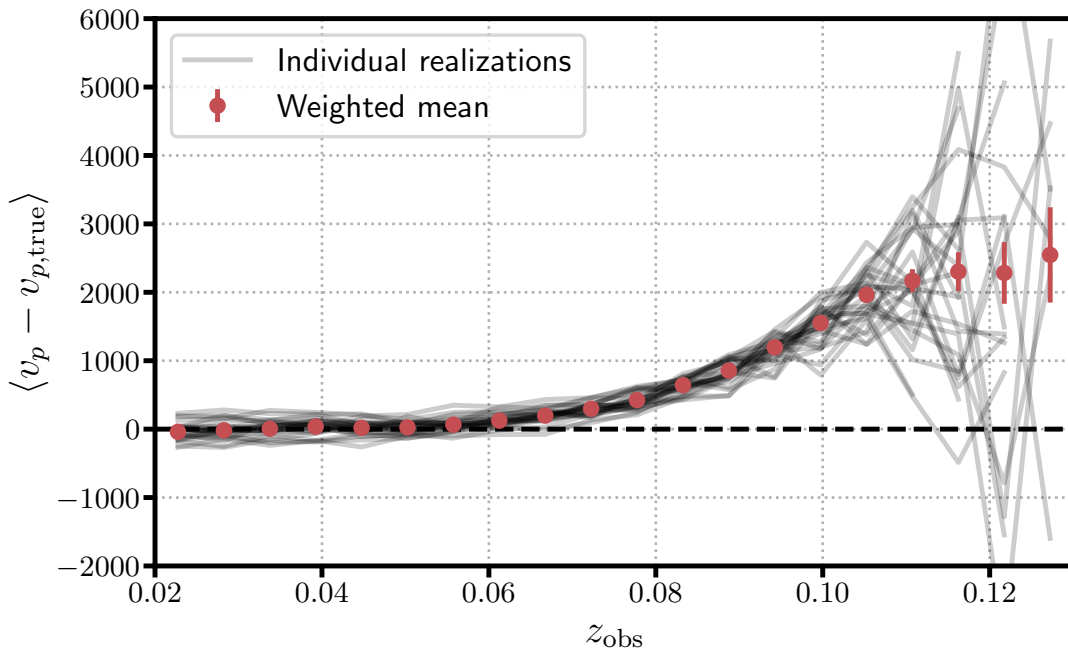


Figure 5.18 – Average difference between estimated velocities and true velocities as a function of *redshift*. The gray lines represent each of the 27 realizations individually, the red points represent the average over all SNe Ia.

Unbiased estimation of velocities can only be achieved when the residuals are distributed according to a normal distribution $\Delta\mu \sim \mathcal{N}(0, \sigma)$ with error σ , centered on 0. Such a distribution guarantees that the velocities will on average be estimated with zero bias. The magnitude limit of the selection function breaks this symmetry by preferentially selecting the brightest objects, that is, those with negative residuals. Most velocities will therefore be estimated as positive, resulting in an overestimation of most velocities. In the next chapter we will see how this bias propagates to the measurement of the structure growth rate $f\sigma_8$.

6 Measuring the growth rate of structures

In Chap 2 we introduced the parameters f and σ_8 related to the growth of structures. These two parameters depend on the theory describing gravity through the Poisson equation and can allow us to measure a deviation from general relativity. In this chapter we focus on the analysis that allows us to constrain the product of these two parameters. In Sect. 6.1 we present the existing methods for measuring $f\sigma_8$. Then, in Sect. 6.2, we describe in more detail the maximum likelihood method that we chose to use. Finally, in Sect. 6.3 we give the main results of our analysis with the ZTF survey simulations that were described in Chap. 5.

6.1 Direct measurement methods for $f\sigma_8$

There are several methods to constrain $f\sigma_8$. First, we can distinguish two types of measurements. Indirect measurements, which use the density field as observable and rely on the effects of velocities on it to measure $f\sigma_8$, and direct measurements, which constrain $f\sigma_8$ using a velocity estimator. In Sec. 2.2.2 we introduced the indirect methods for measuring redshift-space distortions. Here we focus on the direct methods for measuring $f\sigma_8$.

6.1.1 Measuring distances with galaxies

Direct methods generally require two observables, a redshift and a distance. Redshifts are generally measured using spectroscopic surveys on galaxy spectra with a precision of the order of $\sigma_z \sim 10^{-5}$. We saw in Chap. 3 that SNe Ia are very good distance indicators, however given the existing statistics they had not, until now, been used for $f\sigma_8$ measurements. The distance indicators used were instead two empirical relations for galaxies, which we present below.

The Tully-Fisher relation

The Tully-Fisher relation for spiral galaxies is an empirical law relating the rotation velocity of the galaxy to its luminosity (Tully and Fisher 1977)

$$M_b = \alpha \log V_{\max} + \beta, \quad (6.1)$$

where M_b is the absolute magnitude in a band b and α and β are the parameters of the relation to be determined.

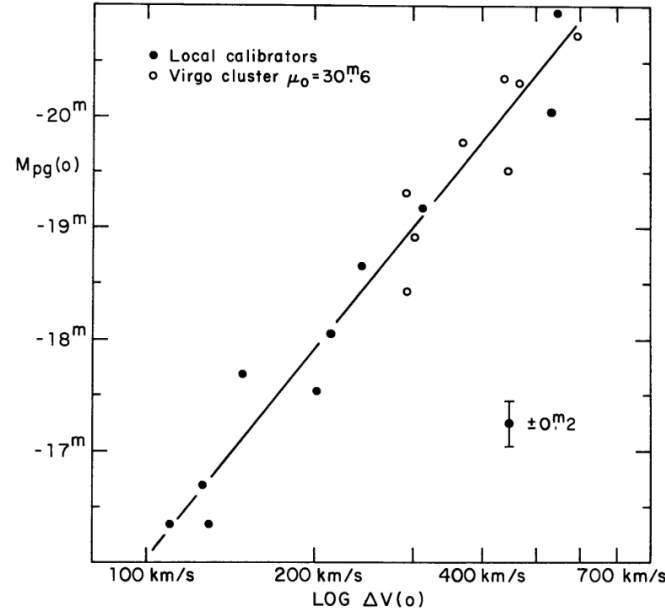


Figure 6.1 – Tully-Fisher relation between the magnitude and the rotation velocity of spiral galaxies. Credit: Tully and Fisher 1977

Distances obtained from the Tully-Fisher (TF) relation can be used in peculiar velocity surveys at very low redshift. Among the catalogs we can cite SFI++ (Springob et al. 2009), CosmicFlows (Kourkchi et al. 2020) or 2MTF (Hong et al. 2019).

The fundamental plane relation

The fundamental plane (FP) relation allows one to estimate the distance of an elliptical galaxy. It is expressed as

$$\log R_e = a \log \sigma + b \log I_e + c, \quad (6.2)$$

where R_e is the effective radius of the galaxy, σ the central velocity dispersion of the galaxy, and I_e the effective surface luminosity. The distance is obtained by comparing the angular diameter from Eq. 6.2 with the observed one.

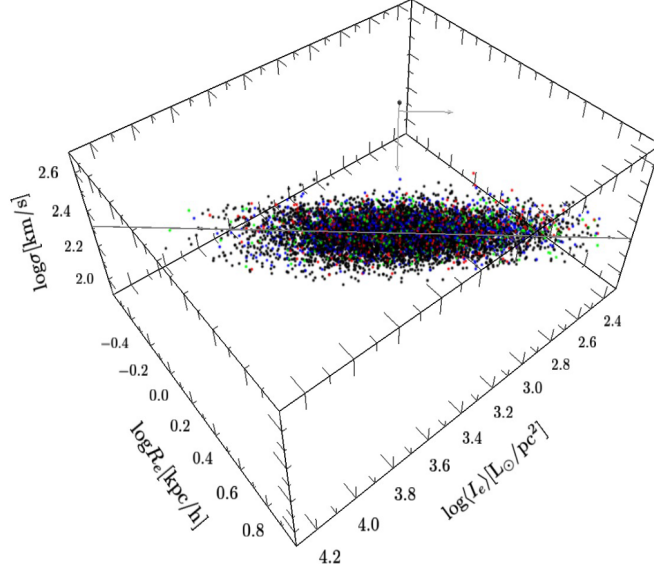


Figure 6.2 – Fundamental plane relation between the effective radius, the surface luminosity and the velocity dispersion of elliptical galaxies. Credit: Magoulas et al. 2012

Among the catalogs used for peculiar velocity analyses are those of the 6dF Campbell et al. 2014 and SDSS Howlett et al. 2022 surveys.

Comparison with SNe Ia

The Tully-Fisher and fundamental plane relations are not as precise as SNe Ia for measuring distances. Indeed, their relative precision on the distance is $\sigma_D/D \sim 20\%$ while SNe Ia have a precision $\sigma_D/D \sim 7\%$. However, galaxies had been used until now because they are much more numerous, since the TF and FP catalogs cited above generally contain several thousand or tens of thousands of distances at low redshift. New SNe Ia surveys such as ZTF and Vera Rubin will allow SNe Ia to reach a sufficient statistics to be competitive for the measurement of $f\sigma_8$.

6.1.2 Different methods for measuring $f\sigma_8$

There are several ways to constrain $f\sigma_8$ using velocities estimated from the combination of redshifts and distance indicators. We present here the main ones.

Maximum likelihood

This method consists in directly fitting $f\sigma_8$ by maximizing a likelihood function $\mathcal{L}(f\sigma_8)$ whose data vector is made up of the velocity field. This is the method used in this work and it is described in more detail in Sec. 6.2. It is notably used with galaxies in Abate and Erdogdu 2009; Johnson et al. 2014; Howlett et al. 2017a; Lai et al. 2023.

Compression of the 2-point statistic

Here we group the methods that use statistical compressions of the estimated velocity field. This can be the estimation of the correlation function (Nusser 2017; Turner et al. 2022), the "pairwise" velocity statistic (Dupuy et al. 2019), or the estimation of the power spectrum of the momentum $p = (1 + \delta_m)v$ (Qin et al. 2019).

Comparison between observed and reconstructed velocities

The 2M++ catalog (Lavaux and Hudson 2011) provides a reconstruction of the density field of the nearby Universe. It uses galaxies as density tracers, so, combined with Eq. 2.66, it allows one to obtain a reconstruction of the velocity field

$$\mathbf{v}_{\text{rec}} = \frac{aH\beta}{4\pi} \int \frac{\mathbf{x}' - \mathbf{x}}{|\mathbf{x}' - \mathbf{x}|^3} \delta_g(\mathbf{x}') d^3\mathbf{x}', \quad (6.3)$$

where we recall that the parameter $\beta = f/b$ expresses the degeneracy between f and the galaxy bias b . By comparing the reconstructed velocities with velocities estimated by TF, FP or SNe Ia, one can constrain β . With an external measurement of $\sigma_{8,g} = b\sigma_8$ one can obtain $f\sigma_8 = \beta\sigma_{8,g}$. This method is used in Pike and Hudson 2005; Carrick et al. 2015; Boruah et al. 2020; Said et al. 2020.

Forward modeling

In the forward modeling method, the density or velocity field itself is a set of free parameters to be fitted. The observables are then modeled from this field to obtain predictions that can be compared to the data. In the case of the velocity field, a first evolutionary forward-modeling framework is described in Lavaux 2016. This model is used to reconstruct the local velocity field in Graziani et al. 2019; Boruah et al. 2021; Prideaux-Ghee et al. 2022. In these works the cosmological parameters are fixed to fiducial values.

6.2 Description of the maximum likelihood method

6.2.1 Likelihood function

In the maximum likelihood method we assume that the velocity field is a Gaussian random field. That is, if we evaluate the velocity field at n positions $\mathbf{r}_i$ in space, the velocities at each point $v_i = v(\mathbf{r}_i)$ are distributed according to an n -dimensional multivariate normal distribution with covariance matrix C . We can then write the likelihood function

$$\mathcal{L}(\mathbf{p}, \mathbf{p}_{\text{HD}}) = (2\pi)^{-\frac{n}{2}} |C(\mathbf{p}, \mathbf{p}_{\text{HD}})|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} \hat{\mathbf{v}}(\mathbf{p}_{\text{HD}})^T C(\mathbf{p}, \mathbf{p}_{\text{HD}})^{-1} \hat{\mathbf{v}}(\mathbf{p}_{\text{HD}}) \right] \quad (6.4)$$

where $\mathbf{p}_{\text{HD}} = \{\alpha, \beta, M_0, \sigma_M\}$ is the vector containing the parameters of the SNe Ia Hubble diagram, and $\mathbf{p} = \{f\sigma_8, \sigma_v, \sigma_u\}$ is the vector containing the parameters related to the velocity field, which are described below.

We seek to minimize $-\log \mathcal{L}$ using a minimization algorithm, or to explore $\mathcal{L}$ around its maximum using a Markov Chain Monte Carlo (MCMC) algorithm.

6.2.2 Data vector

The data vector $\mathbf{v}$ is directly made up of the peculiar velocities. We will see later that we perform tests using directly the "true" velocities of the simulation $v_{\text{true},i}$, as already done in Sec. 5.6. For the estimated velocities we decided to use the estimator $\hat{v}_1$ presented in Eq. 3.60. The parameters Ω_m and H_0 are fixed to our fiducial cosmology.

In the different ZTF realizations, it happens that one to four pairs of SNe Ia are assigned to the same host. To avoid an invalid covariance we group these SNe Ia and assign them a single velocity through an error-weighted average

$$\hat{v}_{\text{eff}} = \frac{\sum \hat{v}_i \sigma_{\hat{v},i}^{-2}}{\sum \sigma_{\hat{v},i}^{-2}}. \quad (6.5)$$

The error on this velocity is

$$\sigma_{v_{\text{eff}}} = \frac{1}{\sqrt{\sum_i \sigma_{\hat{v},i}^{-2}}} \quad (6.6)$$

6.2.3 Modeling the velocity covariance

The velocity covariance $C(\mathbf{p}, \mathbf{p}_{\text{HD}})$ of Eq. (6.4) can be decomposed into several terms

$$C_{ij}(\mathbf{p}, \mathbf{p}_{\text{HD}}) = C_{ij}^{vv}(f\sigma_8, \sigma_u) + \sigma_v^2 \delta_{ij}^K + \sigma_b^2(\mathbf{p}_{\text{HD}}) \delta_{ij}^K. \quad (6.7)$$

The first term is related to the covariance of the cosmological velocity field. We saw in Sec. 2.1.4 the equations related to the velocity field. To obtain the covariance matrix of the velocities projected along the line of sight $\hat{\mathbf{n}}$ between two galaxies i and j , we can use the expression for the velocity in Fourier space from Eq. 2.70. We then get the covariance

$$C_{ij}^{vv} = \langle (\mathbf{v}_i \cdot \hat{\mathbf{n}}_i) (\mathbf{v}_j \cdot \hat{\mathbf{n}}_j) \rangle \quad (6.8)$$

$$= (aHf)^2 \iiint_{-\infty}^{+\infty} \frac{d^3 \mathbf{k}}{(2\pi)^3} \iiint_{-\infty}^{+\infty} \frac{d^3 \mathbf{k}'}{(2\pi)^3} \frac{(\mathbf{k} \cdot \hat{\mathbf{n}}_i)(\mathbf{k}' \cdot \hat{\mathbf{n}}_j)}{k^2 k'^2} \langle \theta_i(\mathbf{k}) \theta_j^*(\mathbf{k}') \rangle e^{i\mathbf{k} \cdot \mathbf{x}_i} e^{-i\mathbf{k}' \cdot \mathbf{x}_j}. \quad (6.9)$$

Applying the definition of the power spectrum from Eq. 2.12 for the variable θ , we get

$$C_{ij}^{vv} = \frac{(aHf)^2}{(2\pi)^3} \iiint_{-\infty}^{+\infty} d^3 \mathbf{k} \frac{(\mathbf{k} \cdot \hat{\mathbf{n}}_i)(\mathbf{k} \cdot \hat{\mathbf{n}}_j)}{k^4} P_{\theta\theta}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{x}_i - \mathbf{x}_j)}. \quad (6.10)$$

We can assume that the power spectrum is isotropic and rewrite Eq. 6.10 in spherical coordinates

$$C_{ij}^{vv} = \frac{(aHf)^2}{(2\pi)^3} \int_0^{+\infty} dk P_{\theta\theta}(k) \int_0^{2\pi} \int_0^\pi d\Omega (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_i) (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_j) e^{i\mathbf{k} \cdot (\mathbf{x}_i - \mathbf{x}_j)}, \quad (6.11)$$

where $\hat{\mathbf{k}} = \frac{\mathbf{k}}{k}$. One can show that the angular integral gives the function W_{ij} such that (Appendix A of Ma et al. 2011)

$$W_{ij}(k; \mathbf{x}_i, \mathbf{x}_j) = \int_0^{2\pi} \int_0^\pi \frac{d\Omega}{4\pi} (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_i) (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_j) e^{i\mathbf{k} \cdot (\mathbf{x}_i - \mathbf{x}_j)} \quad (6.12)$$

$$= \frac{1}{3} \cos \theta_{ij} (j_0(kr_{ij}) - 2j_2(kr_{ij})) + r_{ij}^{-2} j_2(kr_{ij}) x_i x_j \sin^2 \theta_{ij}. \quad (6.13)$$

where $r_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|$, $\theta_{ij} = \hat{\mathbf{x}}_i \cdot \hat{\mathbf{x}}_j$ and j_0 and j_2 are the spherical Bessel functions such that

$$j_0(x) = \frac{\sin x}{x}, \quad (6.14)$$

$$j_2(x) = \left(\frac{3}{x^2} - 1 \right) \frac{\sin x}{x} - 3 \frac{\cos x}{x^2}. \quad (6.15)$$

We have plotted $W(k)$ in Fig. 6.3 for different spatial and angular separations. For two objects at the same position we obtain the expected value for the diagonal terms $W_{ii} = \frac{1}{3}$.

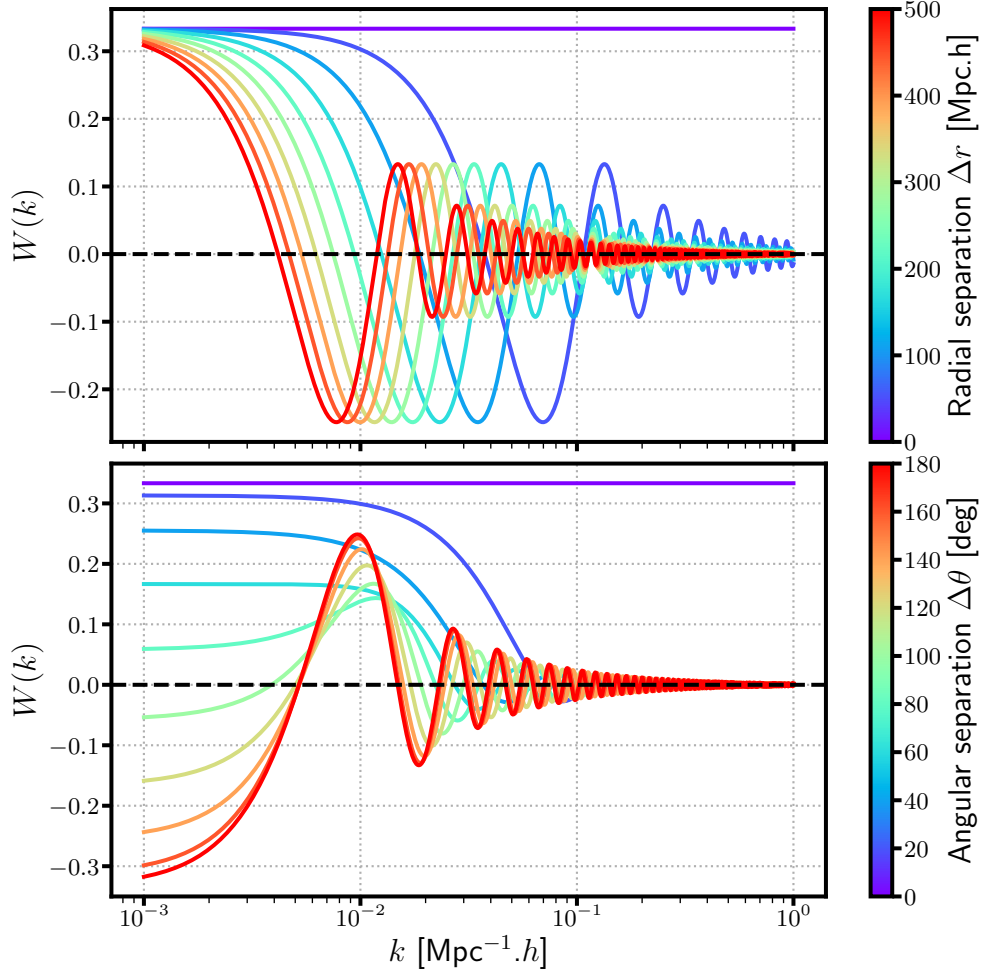


Figure 6.3 – Window function $W(k; \mathbf{x}_i, \mathbf{x}_j)$ as a function of k . *Top*: The colors represent different variations of radial separations with $\theta_{ij} = 0$, $|\mathbf{x}_i| = 200 \text{ Mpc}.h^{-1}$ and $|\mathbf{x}_j| = |\mathbf{x}_i| + \Delta r$. *Bottom*: The colors represent different variations of angular separations with $|\mathbf{x}_i| = |\mathbf{x}_j| = 200 \text{ Mpc}.h^{-1}$.

The covariance can then be written

$$C_{ij}^{vv} = \frac{(aHf)^2}{4\pi^2} \int_0^{+\infty} dk P_{\theta\theta}(k) W_{ij}(k; \mathbf{x}_i, \mathbf{x}_j). \quad (6.16)$$

In practice, we do not know the power spectrum $P_{\theta\theta}$. Since here we seek to fit its amplitude, we can model its shape with a fiducial cosmology. This shape corresponds to the normalized power spectrum of Eq. 2.18, so we have

$$P_{\theta\theta}(k) = \left(\frac{\sigma_8}{\sigma_{8,\text{fid}}} \right)^2 P_{\theta\theta,\text{fid}}. \quad (6.17)$$

We then rewrite the covariance

$$C_{ij}^{vv} = \frac{(f\sigma_8)^2}{(f\sigma_8)_{\text{fid}}^2} \frac{(aH)^2}{4\pi^2} \int_0^{+\infty} dk f_{\text{fid}}^2 P_{\theta\theta, \text{fid}}(k) W_{ij}(k; \mathbf{x}_i, \mathbf{x}_j). \quad (6.18)$$

When building the covariance matrix, the comoving distances of $\mathbf{x}_i$ and $\mathbf{x}_j$ are evaluated using the observed redshifts z_{obs} . These redshifts are themselves affected by the peculiar velocities, so space is distorted as explained in Sec. 2.2.2. We must therefore introduce a so-called damping term to account for the effect of these distortions on the power spectrum. This term reduces the amplitude of the power spectrum at small scales where RSD have a significant contribution. In Koda et al. 2014 this term is introduced empirically by comparison with N -body simulations as

$$D_u(k; \sigma_u) = \frac{\sin(k\sigma_u)}{k\sigma_u}, \quad (6.19)$$

where σ_u is a free parameter, with a value $\sigma_u \sim 15 \text{ Mpc} \cdot h^{-1}$. We add this term to the covariance calculation such that

$$C_{ij}^{vv}(f\sigma_8, \sigma_u) = \frac{(f\sigma_8)^2}{(f\sigma_8)_{\text{fid}}^2} \frac{(aH)^2}{4\pi^2} \int_0^{+\infty} dk f_{\text{fid}}^2 P_{\theta\theta, \text{fid}}(k) W_{ij}(k; \mathbf{x}_i, \mathbf{x}_j) D_u^2(k; \sigma_u). \quad (6.20)$$

We note that recent works attempt to give an analytical expression for the redshift-space velocity power spectrum (Dam et al. 2021).

Although only the linear spectrum has an analytical formula, we know it is not valid at scales $k > 0.1 \text{ Mpc}^{-1} \cdot h$. We therefore need to choose a non-linear model for the power spectrum $P_{\theta\theta}$. In this work, we considered two power spectrum models. The first is our choice for our reference analysis. It is a model fitted on an N -body simulation in Bel et al. 2019. The expression for this power spectrum is

$$P_{\theta\theta}^{\text{non-lin}}(k) = P_{\theta\theta}^{\text{lin}}(k) \exp[-k(a_1 + a_2 k + a_3 k^2)], \quad (6.21)$$

where k is in $\text{Mpc}^{-1} \cdot h$ and the coefficients a_i obtained during the fit are

$$\begin{cases} a_1 &= -0.817 + 3.198\sigma_8 \\ a_2 &= 0.877 - 4.191\sigma_8 \\ a_3 &= -1.199 + 4.629\sigma_8 \end{cases}. \quad (6.22)$$

In Sec. 6.3.3, we compare our results with the use of another power spectrum model obtained using a 2-loop expansion in perturbation theory (RegPT, Taruya et al. 2012). These different power spectrum models are shown in Fig. 6.4.

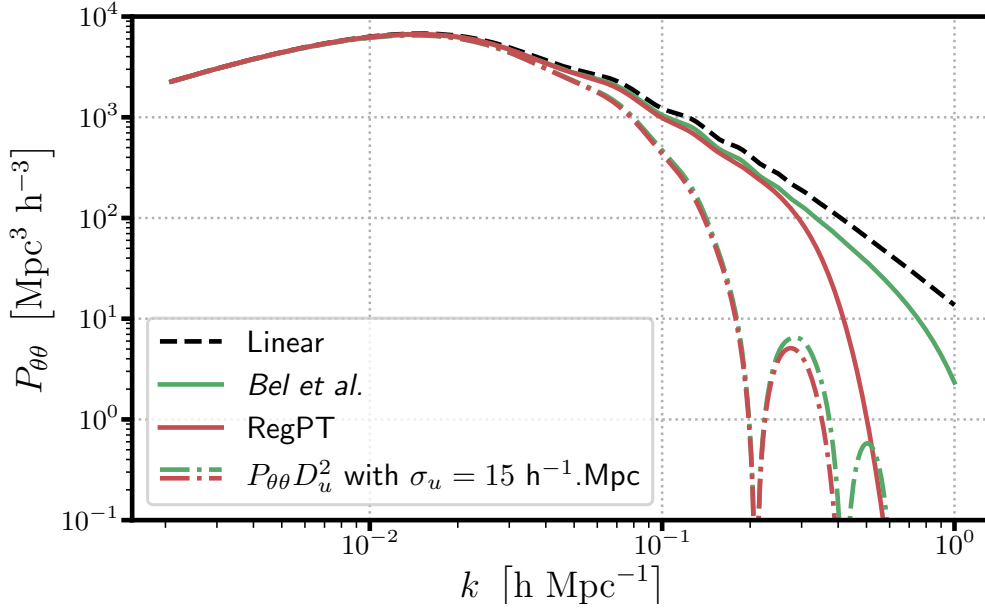


Figure 6.4 – Power spectrum of the velocity divergence parameter θ . The linear model is shown as black dotted lines, the model of Bel et al. 2019 in red, and the RegPT model of Taruya et al. 2012 in green. The dash-dotted lines include the damping factor D_u .

The second term of Eq. 6.7, σ_v , is a free parameter that accounts for the effects of non-linearities on the velocity distribution at small scales. This modeling is imperfect, but there is not yet a power spectrum model that can correctly describe small scales.

The last term of Eq. 6.7, $\sigma_{\hat{v}}$, is the error on the velocity estimator described in Eq. 3.69. We note that $\sigma_{\hat{v}}$ depends on the parameters of the SNe Ia Hubble diagram such as M_0 , α , β , and σ_M .

6.2.4 Covariance for binned velocities

A variation on the previous method is to group the data into bins according to their position in space. This method is used in Johnson et al. 2014 and Howlett et al. 2017a for example, and is inspired by what is done in density field analyses. It consists in choosing a grid size L and averaging the velocities within each cubic cell of this grid. The value of the field in each bin at position $\mathbf{x}_i^G$ is

$$v(\mathbf{x}_i^G) \equiv v_i^G = \frac{\sum_j \hat{v}_j \Theta_{ij}}{\sum_j \Theta_{ij}}, \quad (6.23)$$

where Θ_{ij} is a function indicating the weight of velocity i in bin j . The simplest example is

$$\Theta_{ij} = \begin{cases} 1 & \text{if } |x_i - x_j| < \frac{L}{2}, |y_i - y_j| < \frac{L}{2} \text{ and } |z_i - z_j| < \frac{L}{2}, \\ 0 & \text{otherwise} \end{cases}, \quad (6.24)$$

which corresponds to averaging the velocities contained in each cell, so within cell c_i we have

$$v_i^G = \frac{1}{N_i} \sum_{\mathbf{x}_j \in c_i} \hat{v}_j; \quad (6.25)$$

where N_i is the number of velocities in c_i . The associated error is

$$\sigma_{v^G, i}^2 = \frac{1}{N_i} \sum_{\mathbf{x}_j \in c_i} \sigma_{\hat{v}, j}^2. \quad (6.26)$$

It can be shown that using a grid corresponds to a convolution of the velocity field with it. In Fourier space, this must be accounted for by multiplying the power spectrum by a window function $\Gamma^2(k)$. In the case of the grid expressed by Eq. 6.24, this function is given by

$$\Gamma(k) = \int \frac{d\Omega}{4\pi} \text{sinc}\left(k_x \frac{L}{2}\right) \text{sinc}\left(k_y \frac{L}{2}\right) \text{sinc}\left(k_z \frac{L}{2}\right), \quad (6.27)$$

where $d\Omega = \sin\theta d\theta d\phi$, $k_x = \cos\theta \sin\phi$, $k_y = \sin\theta \sin\phi$ and $k_z = \cos\phi$. Fig. 6.5 shows the window function for different grid sizes L . The larger the grid, the more the small scales are smoothed out.

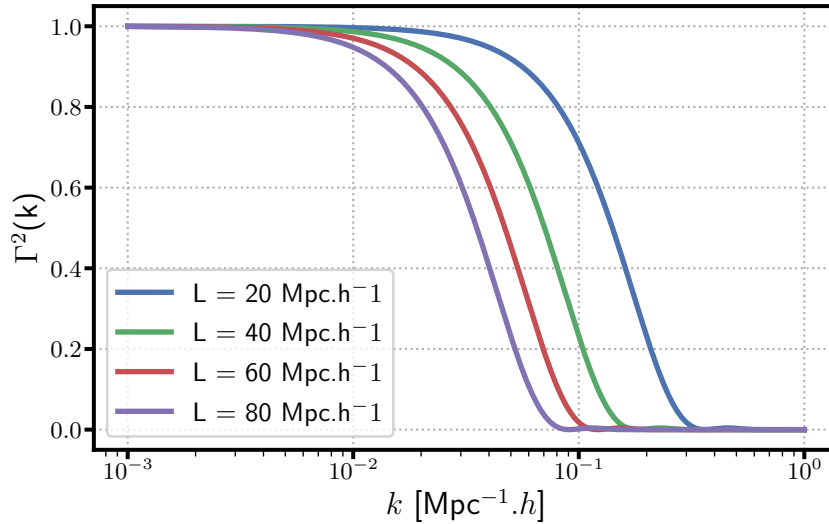


Figure 6.5 – Window function $\Gamma^2(k)$ of the grid used to bin the data. The colors represent $\gamma^2(k)$ for different grid sizes L .

The covariance between cells c_i and c_j is then expressed as

$$\tilde{C}_{ij}^{vv}(f\sigma_8, \sigma_u) = \frac{(f\sigma_8)^2}{(f\sigma_8)_{\text{fid}}^2} \frac{(aH)^2}{4\pi^2} \int_0^{+\infty} dk f_{\text{fid}}^2 P_{\theta\theta}(k) W_{ij}(k; \mathbf{x}_i, \mathbf{x}_j) \Gamma^2(k). \quad (6.28)$$

The damping term D_u is no longer necessary if the grid size is large enough to smooth out the distortions related to peculiar velocities.

This expression for the covariance is no longer valid when the number of data points in a cell becomes small. In Abate and Erdogdu 2009, a correction is proposed for the diagonal terms (Eq. 13)

$$\tilde{C}_{ii}^{vv} \leftarrow \tilde{C}_{ii}^{vv} + \frac{C_{ii}^{vv} - \tilde{C}_{ii}^{vv}}{N_i}. \quad (6.29)$$

It is important to note that in Fig. 3 of Abate and Erdogdu 2009, we see that the correction only seems to have a significant effect when analyzing simulated data without errors.

6.2.5 Power spectrum integration, numerical precision

Computing the covariance matrix of Eqs. 6.20 and 6.28 requires performing a numerical integration over the power spectrum. We describe here the choices made for this integration.

Integration bounds

The choice of the lower integration bound k_{min} is imposed by the size of the N-body simulation. Indeed, N-body simulations generally use periodic boundary conditions. These conditions impose that the modes k with non-zero power are greater than the fundamental mode $k_F = \frac{2\pi}{L}$, where L is the box size. For OuterRim, the box size is $L = 3 \times 10^3 \text{ Mpc}.h^{-1}$, so the fundamental frequency is $k_F \sim 2.1 \times 10^{-3} \text{ Mpc}^{-1}.h$ and our lower integration bound is $k_{\text{min}} = k_F$. In the case of an analysis with real data we would take k_{min} as small as possible. In Johnson et al. 2014, Howlett et al. 2017a and Lai et al. 2023 the upper integration bound is set so as to obtain an unbiased result for $f\sigma_8$ during the tests of the analysis on simulations. This choice can be motivated by cutting the power spectrum before entering a non-linear regime for which it is no longer valid; the limit set is generally $k_{\text{max}} = 0.2 \text{ Mpc}^{-1}.h$. In our work, we opted for a different choice: we integrate the power spectrum with an upper bound such that the integral converges. We have plotted in Fig. 6.6 the integral of the power spectrum as a function of k_{max} for several values of the damping parameter σ_u . We see that for a value of $k_{\text{max}} = 1 \text{ Mpc}^{-1}.h$ the integral of the power spectrum converges for all values of σ_u . This is therefore the choice we use.

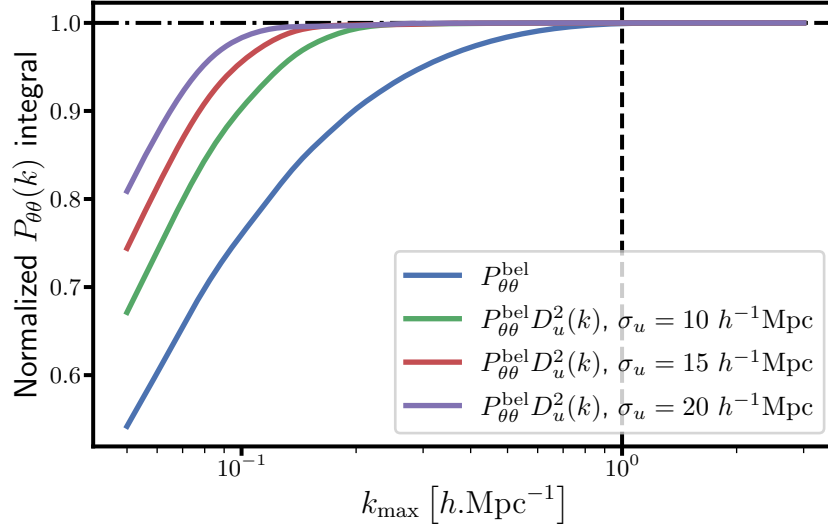


Figure 6.6 – Integral of the power spectrum as a function of the integration bound $k_{\max}$. The different lines represent the integral for different values of the damping parameter σ_u . The integral is normalized with respect to that at $k_{\max} = 10 \text{ Mpc}^{-1}.h$

We note that the choice of $k_{\max} = 0.2 \text{ Mpc}^{-1}.h$ allows the integral to converge for $\sigma_u \sim 15 \text{ Mpc}.h^{-1}$.

Power spectrum sampling

We use the CAMB library to compute our linear power spectrum. To be able to integrate the power spectrum, we must choose a sampling in k . This sampling must be sufficiently precise without requiring too long a computation time for the covariance matrix. We plotted in Fig. 6.7 the variations of the power spectrum as a function of the modes k relative to a sampling of $N = 10^7$. We see that with a sampling of $N = 1000$ the error on the power spectrum is already below 0.1%. If we look at the integral of the power spectrum, it varies only by $\sim 10^{-4}\%$. We therefore choose to work with a sampling of $N = 1000$.

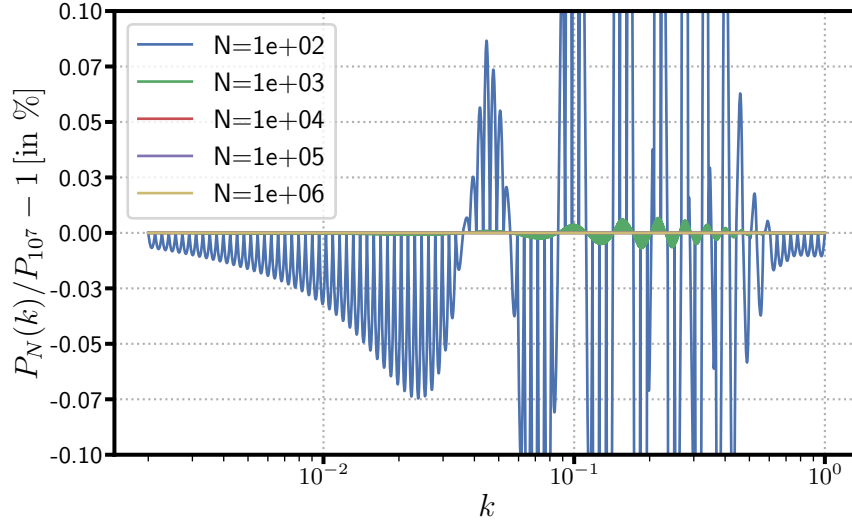


Figure 6.7 – Variation of the power spectrum as a function of k relative to a sampling of $N = 10^7$. The colors correspond to different values of the sampling in k

We can check that the sampling of the power spectrum with $N = 1000$ points is sufficient when we add the window function $W(k)$. Fig. 6.8 shows the relative variation of the integral of the product of the power spectrum with the window function between the samplings $N = 1000$ and $N = 10^6$. This variation is plotted as a function of the window function parameters as presented previously in Fig. 6.3. We can see that the integral does not vary by more than 0.02% between the two samplings.

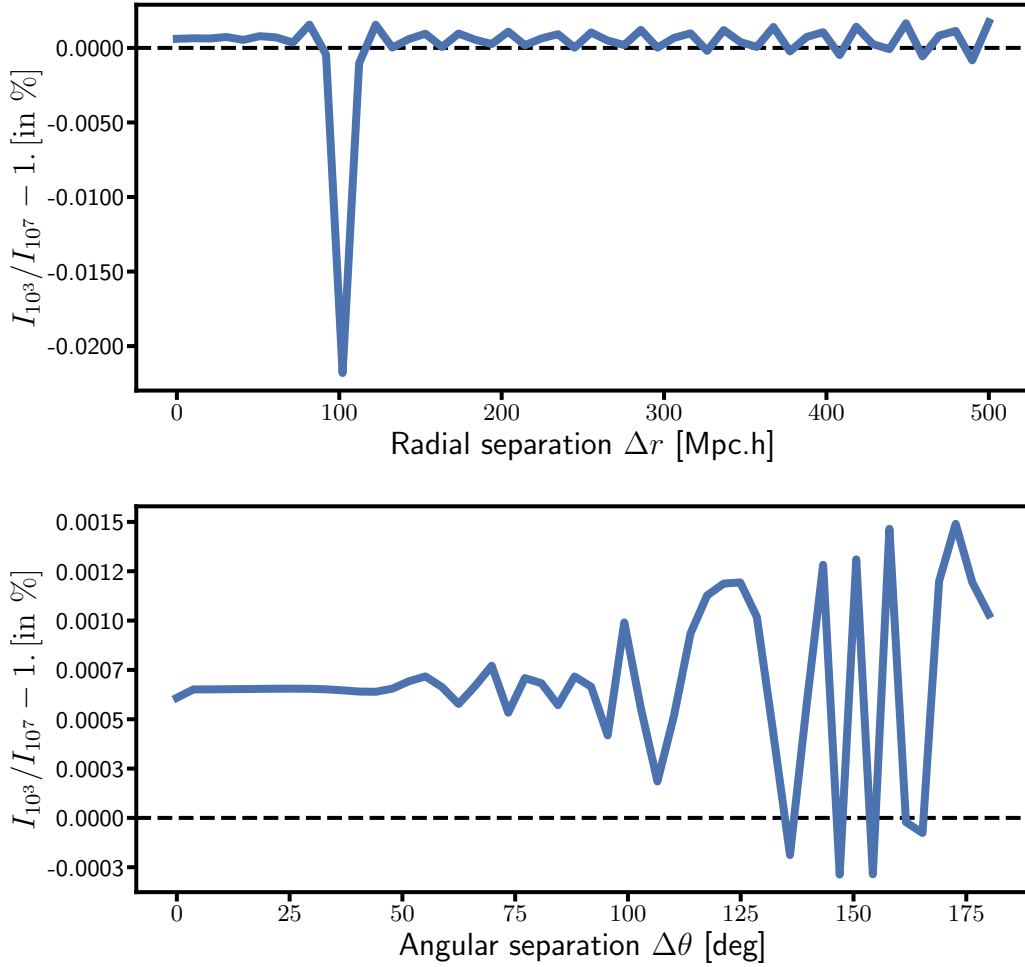


Figure 6.8 – Ratio of the integral $I = \int P(k)W(k;\mathbf{x}_i,\mathbf{x}_j)dk$ of the power spectrum and the window function $W(k;\mathbf{x}_i,\mathbf{x}_j)$ between a sampling $N = 10^3$ and $N = 10^7$. *Top:* The ratio is plotted as a function of the radial separation δr such that $\|\mathbf{x}_j\| = \|\mathbf{x}_i\| + \Delta r$ and with an angular separation $\theta_{ij} = 0$. *Bottom:* The ratio is plotted as a function of the angular separation $\Delta\theta$ and with $\|\mathbf{x}_i\| = \|\mathbf{x}_j\| = 200 \text{ Mpc.}h^{-1}$.

6.2.6 Tests on simple catalogs

To perform a first check of the method, we simulate catalogs as follows:

1. We draw N angular positions (RA, Dec) uniformly on the sphere.
2. We generate N redshifts following the probability density law $\mathcal{P}(z) = \frac{3r^2(z)}{V(z_{\max}) - V(z_{\min})}$.
3. We compute the covariance matrix $C^{\nu\nu}$ and a power spectrum $P(k)$.
4. We draw velocities according to the multivariate normal law $\mathcal{N}(\mathbf{0}, C^{\nu\nu})$.

This yields a catalog of velocities that corresponds perfectly to the power spectrum $P(k)$ used for the simulation.

In Fig. 6.9 we show the results of fitting these catalogs under conditions with and without a grid, and with and without the correction of Abate and Erdogdu 2009 (Eq. 6.29). In this fit, we do not account for redshift-space distortions since they are not present in these simplified simulations. In the top panel of Fig. 6.9, we plot the results for a fit where σ_v is fixed to $\sigma_v = 0$. The result without a grid is recovered without bias. For the fits with a grid, we find results similar to Fig. 3 of Abate and Erdogdu 2009. In the case without correction, the results are biased by more than $\sim 20\%$, and this bias increases as the number of SNe Ia per cell decreases with a smaller grid size. With the correction we also obtain slightly biased results, $\sim -3\%$ for a $40 \text{ Mpc}.h^{-1}$ grid and $\sim 4\%$ for a $100 \text{ Mpc}.h^{-1}$ grid. In the bottom panel we produced the same fit while leaving σ_v free. In the case without a grid, this biases the result very slightly ($\sim -0.02\%$). The results with a grid are modified compared to the previous fit: the bias in the case without correction is of the order of $\sim -1.5\%$ and that in the case with the correction is of the order of $\sim -4\%$.

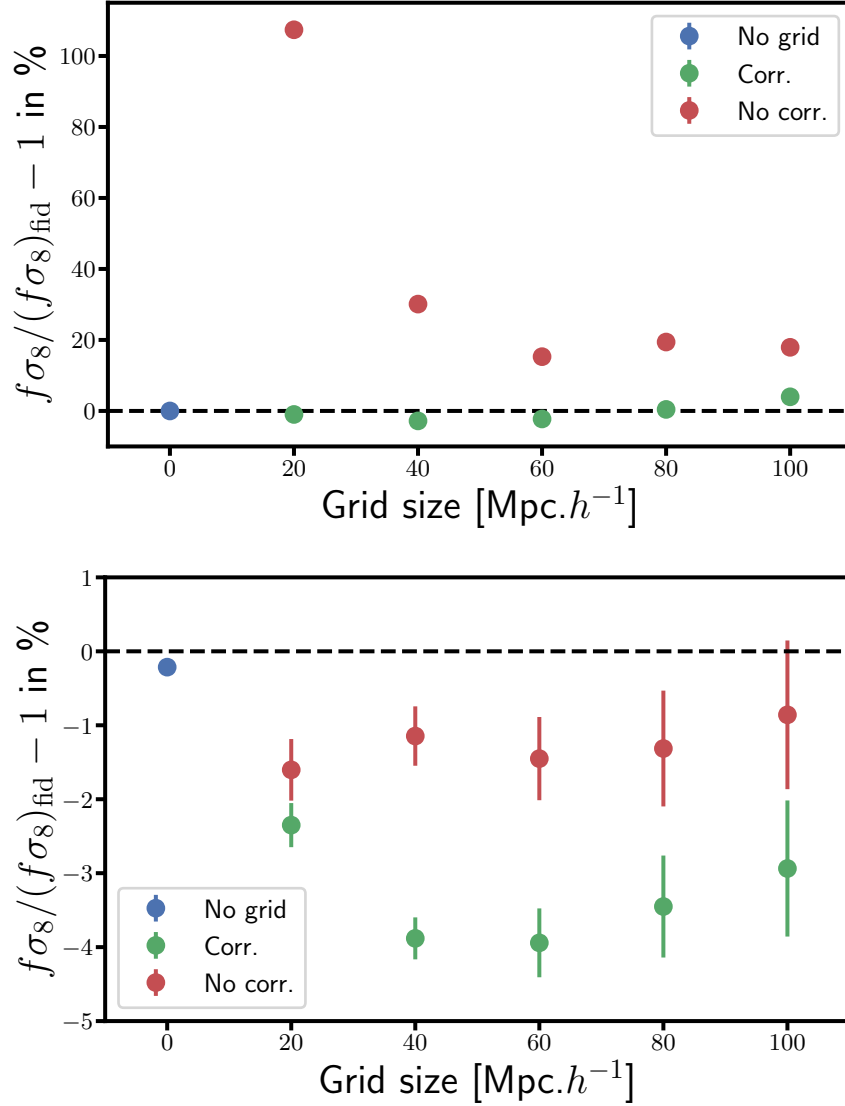


Figure 6.9 – Fits of the simplified velocity field simulations. We compare the fit without a grid (in blue), the one with a grid and with the correction of Abate and Erdogdu 2009 (in green), and the one with a grid and without the correction (in red). *Top:* Fit with σ_v fixed to the value $\sigma_v = 0$. *Bottom:* Fit with σ_v free.

During the analysis of the fit with a grid, we noticed that, in the case of the ZTF survey, it is difficult to find a balance between the number of SNe Ia per cell and the cell size. Indeed, having cells that are too large decreases the sampling of the velocity field. Moreover, the velocity field along the line of sight is not a "true" scalar field, in the sense that, when averaging velocities within a cell, one must assume that these velocities are parallel. This assumption becomes less and less valid as the cell size increases. In Howlett et al. 2017a, for example, a fit with a grid is performed, but the number of data points is larger, with ~ 2062 galaxies, and in a smaller volume since

the redshift range is $0 < z < 0.03$. In the case of our ZTF simulation we have fewer than 2000 SNe Ia in a larger volume $0 < z < 0.06$. Given the difficulties encountered with the fit using a grid, and since the fit without a grid seemed to give us more reliable results, we decided to focus on the latter.

6.3 Main results

We applied the maximum likelihood method to the ZTF simulations described in Chap. 5. For statistical reasons, the realizations equivalent to six years of ZTF are used in the studies that follow, with the exception of the prediction for the ZTF DR2 (Sec. 6.3.5). We present in this section the main results published in Carreres et al. 2023. In this work we took into account the errors on the distance moduli obtained with the SNe Ia, but we neglected the errors on the redshift. We verified that, in the framework of redshifts obtained by spectroscopy with a measurement error $\sigma_z \sim 10^{-5}$, this error has a negligible impact on $f\sigma_8$.

6.3.1 Predicted constraints on $f\sigma_8$ for six years of ZTF

As a first approach, we can avoid the selection bias by keeping only the SNe Ia with a redshift $z < 0.06$. This dataset, called complete, reduces the statistics to $\langle N_{\text{SN}} \rangle \sim 1620$, which represents half of the simulated SNe Ia.

To study the constraint on $f\sigma_8$, we performed three different fits. These fits are performed by minimizing the quantity $-\log \mathcal{L}$, where $\mathcal{L}$ is the likelihood function of Eq. 6.4, using the Minuit algorithm already used for the light-curve fits in Sec. 5.6.2.

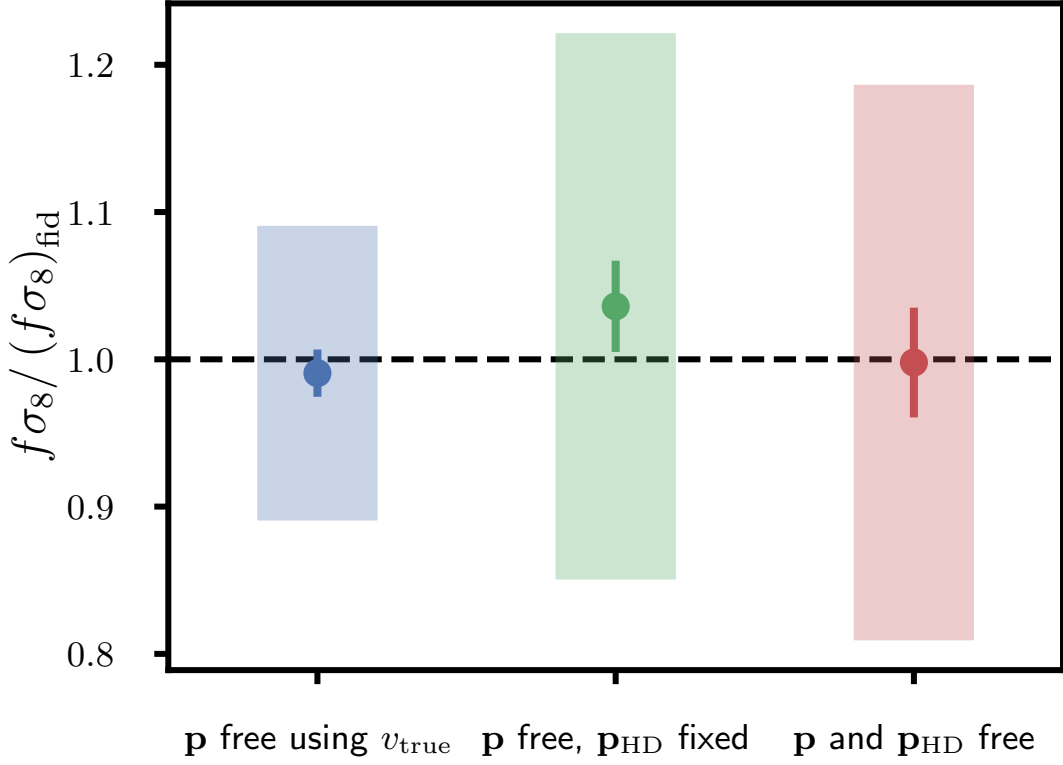


Figure 6.10 – Result of the fit of $f\sigma_8$ for the SNe Ia classified spectroscopically in the redshift interval $z_{obs} \in [0.02, 0.06]$. The points with error bars show the mean obtained over our 27 realizations, the colored rectangles represent the average uncertainty on $f\sigma_8$.

The first fit performed is that of the true velocities, i.e., those output by the N -body simulation. The result of this fit is shown in blue in Fig. 6.10. Over our 27 realizations we obtain the result

$$\langle f\sigma_8 / (f\sigma_8)_{\text{fid}} \rangle = 0.991 \pm 0.016. \quad (6.30)$$

This result is compatible within the $1\text{-}\sigma$ error bar with the fiducial value of the simulation. This shows that the maximum likelihood model is valid. The average uncertainty over the 27 realizations is given by

$$\sqrt{\langle \sigma_{f\sigma_8}^2 \rangle} / (f\sigma_8)_{\text{fid}} = 0.100. \quad (6.31)$$

Since this measurement is performed in a case where the velocities are perfectly known, this uncertainty represents the lower limit of the error that can be obtained for $f\sigma_8$ with the statistics of the simulation.

In the second fit, we use the estimated velocities $\hat{v}_i$, but we fix the Hubble diagram parameters $\mathbf{p}_{\text{HD}}$ to their simulated values. During the first attempts of this fit, we observed that the statistics were not large enough to constrain σ_u . The degeneracy between high values of σ_u and $f\sigma_8$ then leads to an overestimation of σ_u and $f\sigma_8$. To

Table 6.1 – Results obtained for the parameters $p \in \mathbf{p}, \mathbf{p}_{\text{HD}}$ with our 27 realizations of 6 years of the ZTF SNe Ia survey in a redshift interval $z \in [0.02, 0.06]$.

Parameters	p_{true}	$\langle p \rangle$	$\sqrt{\langle \sigma_p^2 \rangle}$
$f\sigma_8 / (f\sigma_8)_{\text{fid}}$	1.0	0.998 ± 0.037	0.188
α	0.14	0.1356 ± 0.0006	0.004
β	3.1	3.054 ± 0.006	0.04
M_0	-19.019	-19.027 ± 0.002	0.014
σ_M	0.12	0.1196 ± 0.0008	0.004
σ_u	-	14.1 ± 0.5	6.4
σ_v	-	168 ± 20	186

compensate for this problem, some works decide to fix σ_u based on simulation results (Lai et al. 2023). In this work, we decide to use a Gaussian prior centered on $\mu_{\sigma_u} = 15 \text{ Mpc}.h^{-1}$ with a width $\sigma_{\sigma_u} = 0.5\mu_{\sigma_u}$. This value is close to what is observed in Koda et al. 2014 and to the result obtained in our first fit. The dependence of $f\sigma_8$ on the prior is discussed in Sec. 6.3.2. The fit is shown in green, in the center of Fig. 6.10. The mean value of $f\sigma_8$ is

$$\langle f\sigma_8 / (f\sigma_8)_{\text{fid}} \rangle = 1.036 \pm 0.031. \quad (6.32)$$

This value is a little more than 1σ from the fiducial value. This possible bias is negligible compared to the average uncertainty

$$\sqrt{\langle \sigma_{f\sigma_8}^2 \rangle} / (f\sigma_8)_{\text{fid}} = 0.185. \quad (6.33)$$

In this fit we can notice on the central panel of Fig. 6.14 that for at least one of the realizations, the value of $f\sigma_8$ obtained seems low. It appears that the fit can become unstable, in particular with a low- z cutoff, where the statistics are weakest.

In the third fit, we vary both the parameters $\mathbf{p}$ and the Hubble diagram parameters $\mathbf{p}_{\text{HD}}$. The results of this fit are shown in red, on the right of Fig. 6.10. The value of $f\sigma_8$ obtained is

$$\langle f\sigma_8 / (f\sigma_8)_{\text{fid}} \rangle = 0.998 \pm 0.037. \quad (6.34)$$

This value is compatible with the fiducial value. As in the previous fit, we can see on the bottom panel of Fig. 6.14 that one of the realizations gives a result for $f\sigma_8$ that seems very low. However, the Minuit minimizer reports the minimum obtained as valid, so we decided to keep this result and check that it did not significantly change the average value of $f\sigma_8$ obtained. The average uncertainty on $f\sigma_8$ for this fit is

$$\sqrt{\langle \sigma_{f\sigma_8}^2 \rangle} / (f\sigma_8)_{\text{fid}} = 0.188. \quad (6.35)$$

We note that the uncertainty does not vary between the case where $\mathbf{p}_{\text{HD}}$ is fixed and the one where $\mathbf{p}_{\text{HD}}$ is free. The results for all the fit parameters are given in Table 6.1. The value of M_0 is ~ 0.01 mag lower than the fiducial value, roughly the same order

of magnitude as the selection bias in the Hubble diagram residuals at $z \sim 0.06$. The standardization parameters α and β are, on average, biased relative to the fiducial values. In the case of a fit on the Hubble diagram before selection, we do recover the input values. These biases may therefore come from selection effects. However, they seem to have a negligible effect on the value of $f\sigma_8$, which can be explained by the absence of strong correlation between α , β and $f\sigma_8$. The intrinsic variance parameter of the SNe Ia, σ_M , is recovered with good precision.

To check whether our model correctly accounts for the error due to observational noise and that due to cosmic variance, we can compare the average uncertainty to the dispersion of the results obtained for $f\sigma_8$ across our 27 realizations. We have in Table 6.1 an average uncertainty on $f\sigma_8$ of 0.188. The standard deviation of $f\sigma_8$ across the different realizations is $\text{STD}(f\sigma_8) \simeq 0.194$. This seems to indicate that our analysis correctly accounts for the different sources of error.

To analyze the likelihood function $\mathcal{L}$ we also used the MCMC chain algorithm from the Python library `emcee`¹. The posteriors of the likelihood function obtained for one of the realizations are plotted in Fig. 6.11. We can observe the agreement between the MCMC chain contours (in red) and the results obtained with `Minuit` (in green). We note that the errors computed by `Minuit` are obtained by the `minos` algorithm, which uses an exploration of the 1D likelihood profiles of each parameter². As stated previously, we do not observe any particular correlation between the parameters α , β , M_0 and $f\sigma_8$. We can notice a correlation between σ_v and σ_M , whose degeneracy is mainly broken by the redshift dependence of σ_v . The asymmetric error bar on $f\sigma_8$ appears slightly wider for high values of $f\sigma_8$. This can be explained by the degeneracy observed for large values of $f\sigma_8$ and σ_u . Moreover, we see that the constraint on σ_u is strongly dominated by our Gaussian prior (dotted blue line).

1. <https://github.com/dfm/emcee>

2. For more details see https://iminuit.readthedocs.io/en/stable/notebooks/hesse_and_minos.html

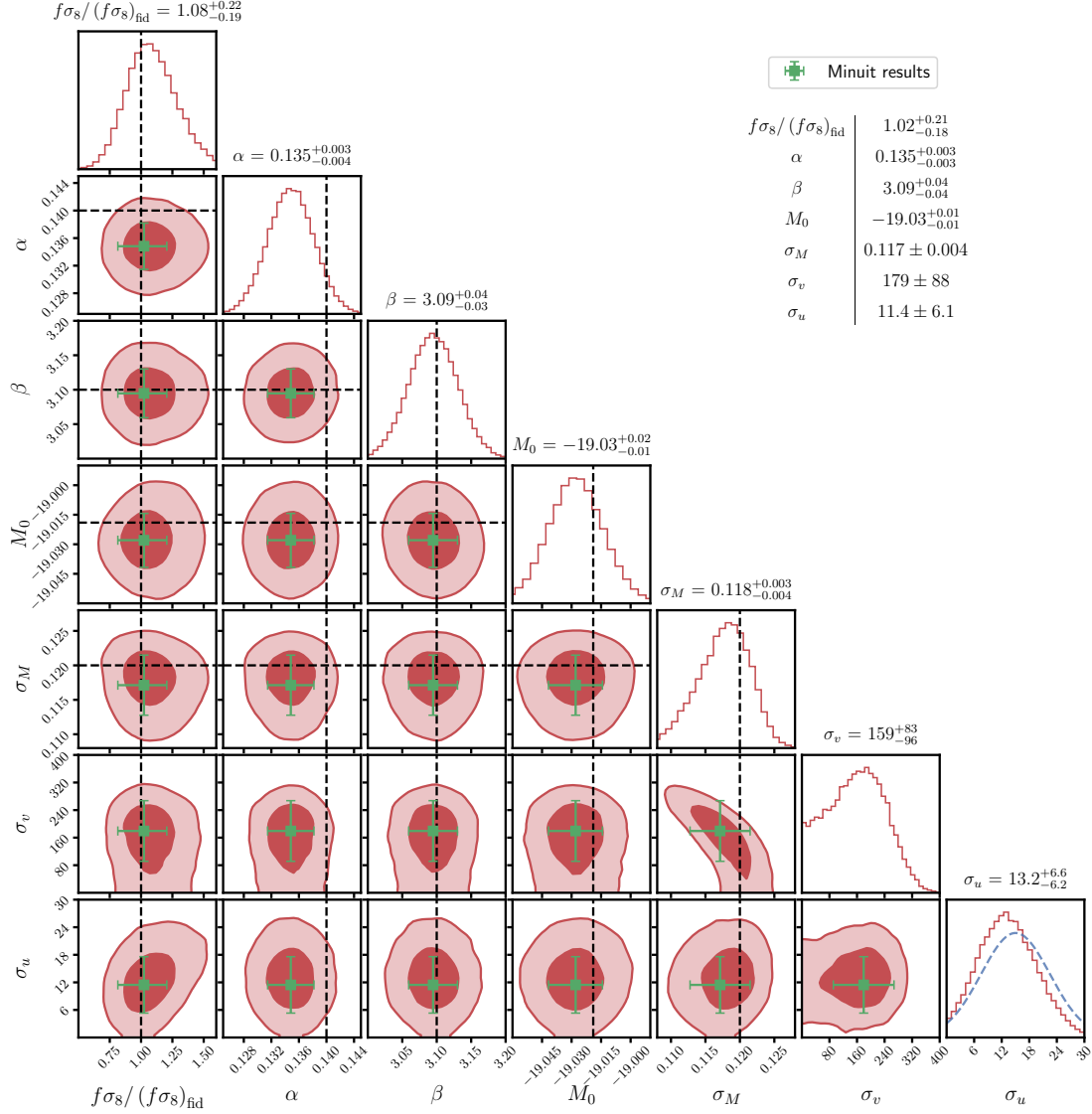


Figure 6.11 – Profile of the likelihood function obtained using MCMC chains during the joint fit of the Hubble diagram and the structure growth parameters for one of our ZTF SNe Ia realizations. The red contours represent the 1- and 2- σ confidence intervals, the black dashed lines indicate the fiducial values of the parameters, the blue dashed line represents the prior on σ_u , and the green squares show the results obtained with Minuit.

6.3.2 Variations of the prior on σ_u

In the analysis presented in the previous section, we saw that it was necessary to give a Gaussian prior for the parameter σ_u . We chose to fix the center and width of this prior to $\mu_{p(\sigma_u)} = 15 \text{ Mpc} \cdot h^{-1}$ and $\sigma_{p(\sigma_u)} = 0.5\mu_{\sigma_u}$. Here we test how this choice influences the result obtained for $f\sigma_8$.

We first vary the central value of the prior, with the width fixed to 50% of $\mu_{p(\sigma_u)}$. The results obtained are plotted in Fig. 6.12. We can see that for $\mu_{p(\sigma_u)} \in [5, 23] \text{ Mpc} \cdot h^{-1}$, the result for $f\sigma_8$ varies from ~ -9 to $\sim +6$ percent relative to our reference result.

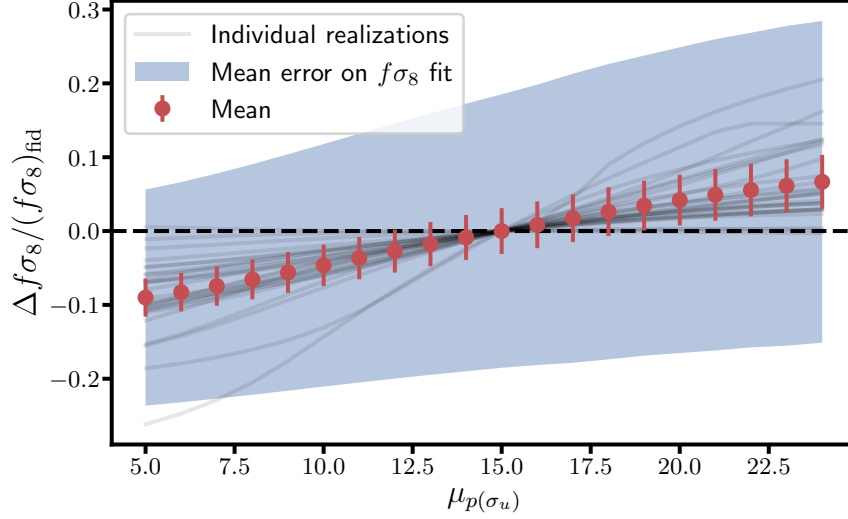


Figure 6.12 – Variation of the result on $f\sigma_8$ as a function of the central value of the prior on σ_u relative to the reference result with $\mu_{p(\sigma_u)} = 15 \text{ Mpc} \cdot h^{-1}$. The red points represent the variation of the mean $f\sigma_8$ for each central value of the prior, the blue band indicates the average uncertainty obtained, and the gray lines indicate the results for each of the 27 realizations.

Our second test consists in fixing the central value of the prior to our reference value $\mu_{p(\sigma_u)} = 15 \text{ Mpc} \cdot h^{-1}$ and varying the width of the prior $\sigma_{p(\sigma_u)}$. The results, shown in Fig. 6.13, show that the fit result for the central value of $f\sigma_8$ is independent of the width of the prior, while the error on $f\sigma_8$ varies between 18% and 20% between a fixed value of σ_u and a prior such that $\sigma_{p(\sigma_u)} / \mu_{p(\sigma_u)} = 1$.

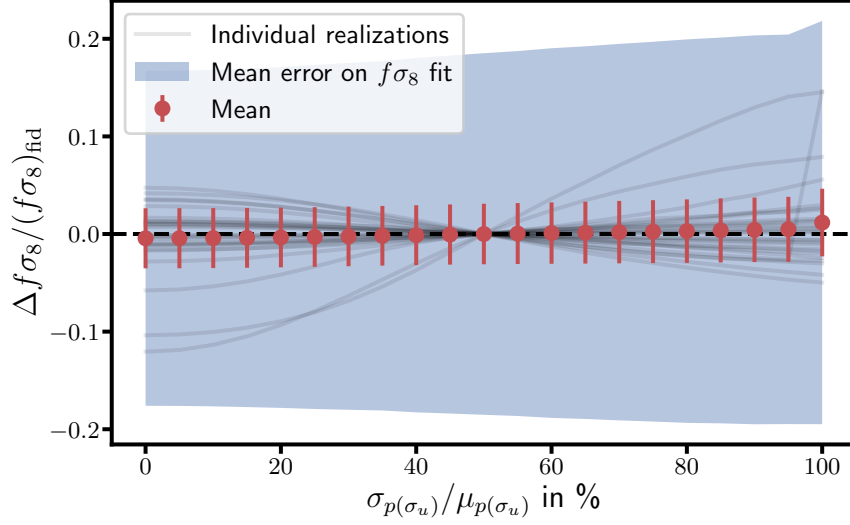


Figure 6.13 – Variation of the result on $f\sigma_8$ as a function of the width of the prior on σ_u relative to the reference result with $\sigma_{p(\sigma_u)} = 0.5\mu_{p(\sigma_u)} \text{ Mpc}.h^{-1}$. The red points represent the variation of the mean $f\sigma_8$ for each width value of the prior, the blue band indicates the average uncertainty obtained, and the gray lines indicate the results for each of the 27 realizations.

6.3.3 Using different power spectrum models

In our reference analysis we used the power spectrum model of Bel et al. 2019. As indicated in Sec. 6.2.3, to test the resilience of our result to this choice, we reproduced the analysis of the true velocities with the power spectrum model of Taruya et al. 2012. Here we use all SNe Ia up to $z = 0.13$, since the true velocities are not biased. We obtain, on average over our 27 realizations, a variation of $f\sigma_8$

$$\langle \Delta f\sigma_8 \rangle / (f\sigma_8)_{\text{fid}} = (-1.5 \pm 4.9) \times 10^{-4}. \quad (6.36)$$

This variation is negligible, which is not surprising since the integrals of the two power spectrum models differ by less than $\sim 1\%$ when $\sigma_u = 15 \text{ Mpc}.h^{-1}$.

A second test we applied is the use of the linear power spectrum in the analysis. The variation of $f\sigma_8$, when using the linear spectrum, is

$$\langle \Delta f\sigma_8 \rangle / (f\sigma_8)_{\text{fid}} = (-2.7 \pm 0.06) \times 10^{-2}. \quad (6.37)$$

This difference of about $\sim 3\%$ can be explained by the fact that the linear power spectrum overestimates the variance of the velocities at small scales (Fig. 6.4). This bias remains small compared to the uncertainty found on $f\sigma_8$ in our predictions for the ZTF analysis.

6.3.4 Using SNe Ia beyond $z > 0.06$

We saw in Sec. 6.3.1 that limiting our data to $z < 0.06$ cuts our statistics in half. Here we look at the consequences of the selection bias on $f\sigma_8$ if we remove the redshift limit, and how the measurement of $f\sigma_8$ evolves with a perfect correction of this bias.

Effect of the selection bias on $f\sigma_8$

In Fig. 6.14, we show the results for $f\sigma_8$ as a function of the redshift limit z_{max} . The top panel shows the results for the fit using the true velocities. We see that the value of $f\sigma_8$ remains unbiased, since, as discussed in Sec. 5.4, the true velocities are not biased by the selection.

The middle panel shows the results for the method where the SNe Ia Hubble diagram parameters are fixed to their input values. We see that at redshifts above $z \simeq 0.06$, the bias observed in the Hubble diagram residuals and the estimated velocities (Figs. 5.17 and 5.18) translates into a positive bias in the value of $f\sigma_8$. The large values of the estimated velocities artificially modify the velocity distribution, which is then no longer a normal distribution centered on zero. This distribution is therefore not correctly modeled by our likelihood function. Since our likelihood function is a Gaussian centered on zero, the positively biased velocities are accounted for by an increase in the width of the distribution, leading to a positive bias on $f\sigma_8$.

In the last panel we have the results for the method where all parameters are free. The bias, although present, seems to kick in slightly later and take smaller values than in the previous case. The parameter M_0 is free here and is affected by the selection bias. It absorbs part of the bias and reduces the large values of the estimated velocities. This is shown in Figs. 6.15 and 6.16. Fig. 6.15 shows the results for M_0 as a function of the cutoff redshift z_{max} ; we see that the value of M_0 progressively decreases as the population of SNe Ia biased by the selection is included in the measurement. In Fig. 6.16, we show the average bias on the velocities as a function of z , for the analyses with different cutoffs z_{max} . The decrease in the value of M_0 leads to a negative bias on the velocities at low redshift and reduces the bias at high redshift, compared to the analysis where M_0 is fixed (Fig. 5.18).

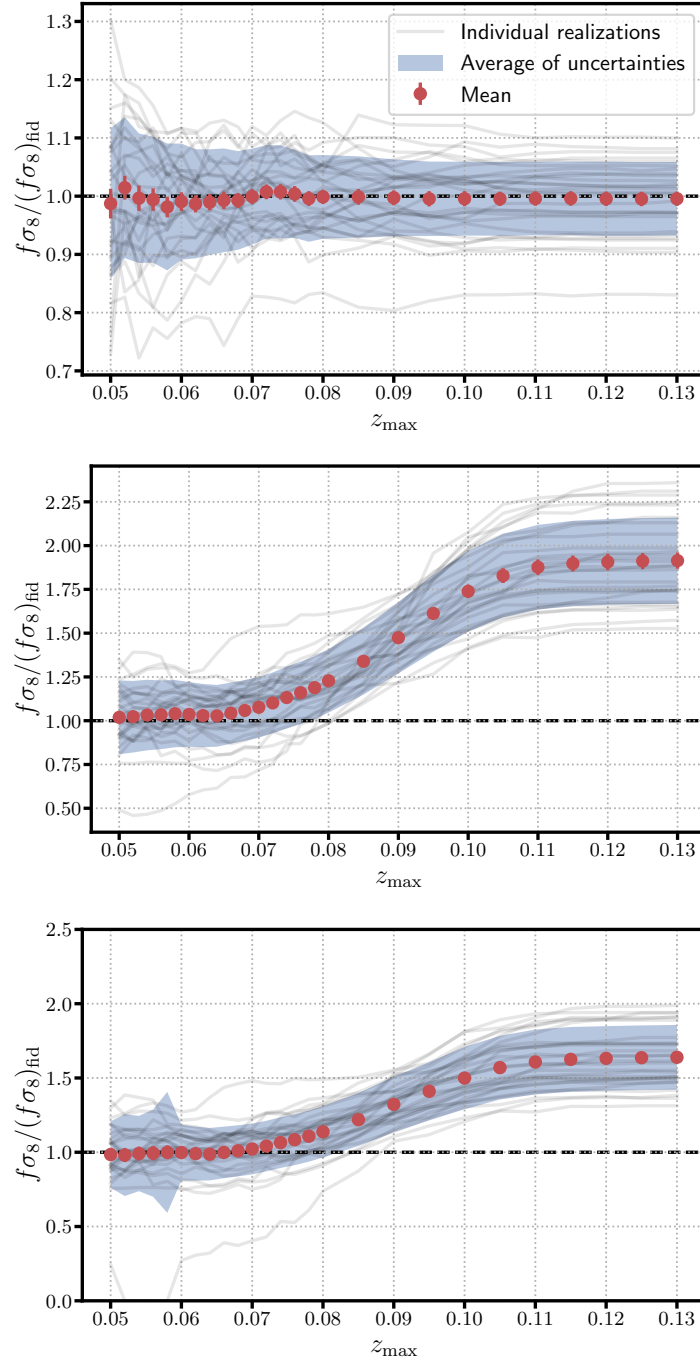


Figure 6.14 – Results of the fit of $f\sigma_8$ as a function of the cutoff redshift z_{max} for the 27 realizations of the dataset equivalent to 6 years of ZTF. The red points represent the mean value of $f\sigma_8$ for each z_{max} , the blue band indicates the average uncertainty obtained on $f\sigma_8$, and the gray lines indicate the results for each of the realizations. *Top:* Results of the fit on the true velocities v_{true} . *Center:* Results of the fit on the estimated velocities $\hat{v}$ with $\mathbf{p}$ free and $\mathbf{p}_{\text{HD}}$ fixed. *Bottom:* Results of the fit on the estimated velocities $\hat{v}$ with $\mathbf{p}$ and $\mathbf{p}_{\text{HD}}$ free.

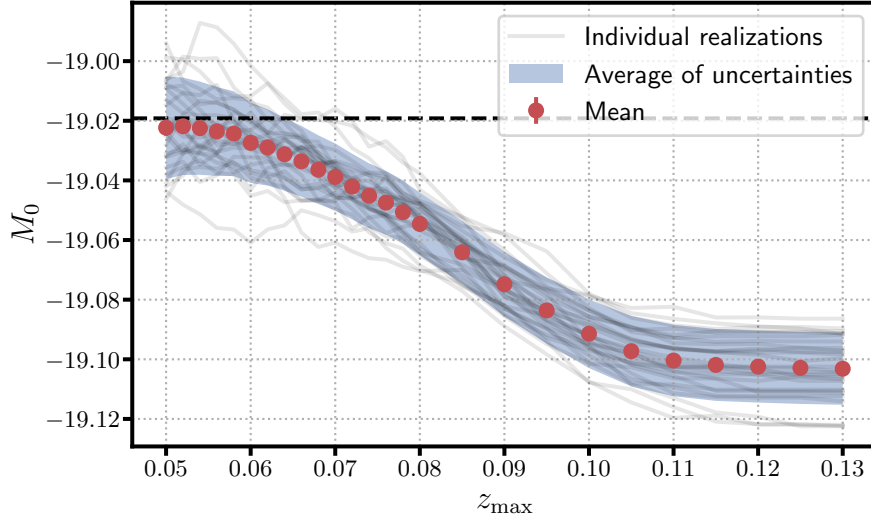


Figure 6.15 – Results obtained for the absolute magnitude of the SNe Ia M_0 as a function of the cutoff redshift $z_{\max}$. The red points represent the mean value of M_0 per redshift interval, the blue band indicates the average uncertainty obtained on M_0 , and the gray lines indicate the results for each of the 27 realizations.

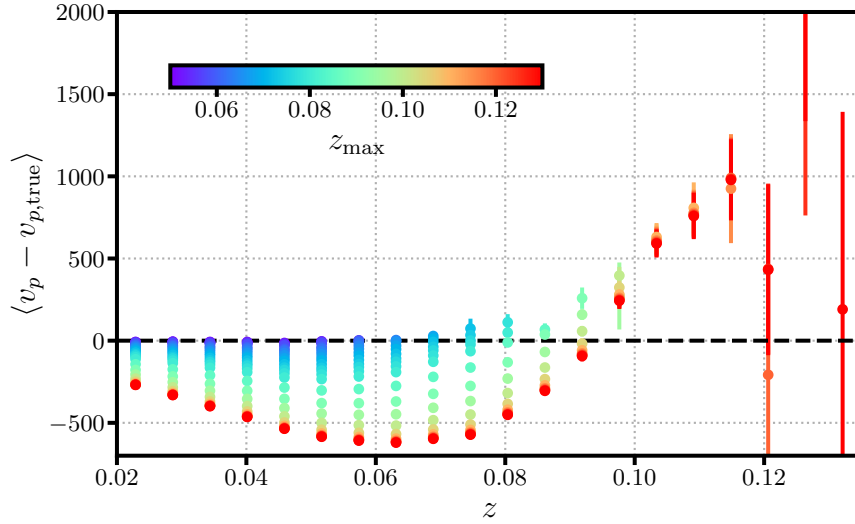


Figure 6.16 – Average bias on the estimated velocities as a function of redshift z . The points indicate the average bias per redshift interval. The color scale varies according to the cutoff redshift $z_{\max}$ used in the fit. The negative bias on M_0 due to the selection bias for cutoffs $z_{\max} > 0.06$ leads to a negative bias on the estimated velocities at low redshift.

Perfect bias correction

The possibility of correcting the selection bias using the methods used in Betoule et al. 2014 or Kessler and Scolnic 2017 is not directly applicable. For example, a correction by redshift shell would, on the one hand, correct the observed distance modulus μ_{obs} to bring it closer to $\mu_{\text{th}}(z_{\text{obs}})$, which should correct the velocity estimate, but on the other hand such a correction could create a spurious correlation between all SNe Ia at the same redshift by adding an identical velocity correction. Another possibility for correcting the bias would be to write a new likelihood function that would model the velocity distribution modified by the SNe Ia selection function (see Kim 2021).

During the work on the ZTF simulation, we decided not to attempt a correction, but rather to quantify the improvement in the measurement that could be obtained with a perfect correction of the bias. To simulate this correction, we can take the values of the true velocity of each SNe Ia v_{true} , which are not biased by the selection (Fig. 5.15), and draw new unbiased velocities from a Gaussian distribution centered on v_{true} with a width equal to the error on the estimated velocity $\sigma_{\hat{v}}$

$$\hat{v}_{\text{debias}}^i \sim \mathcal{N}(v_{\text{true}}^i, \sigma_{\hat{v}}^i). \quad (6.38)$$

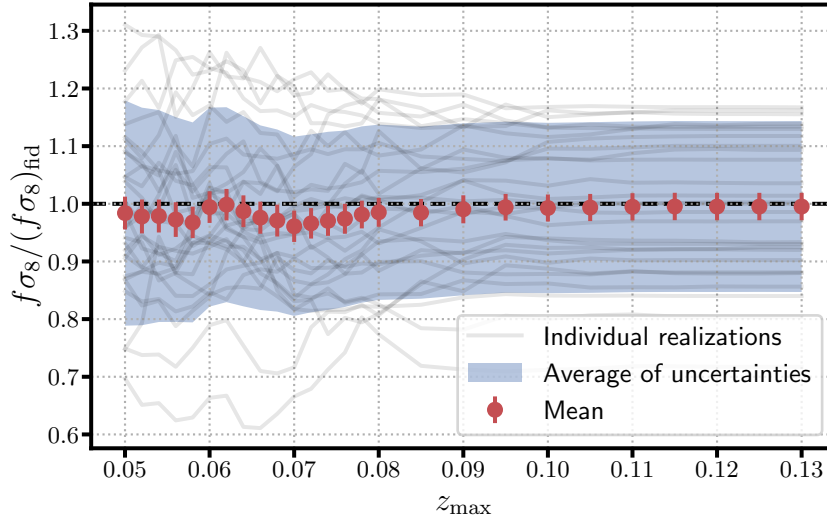


Figure 6.17 – Result of the fit for $f\sigma_8$ with unbiased estimated velocities. The red points represent the mean value of $f\sigma_8$ per redshift interval, the gray lines indicate the results for each of the 27 realizations.

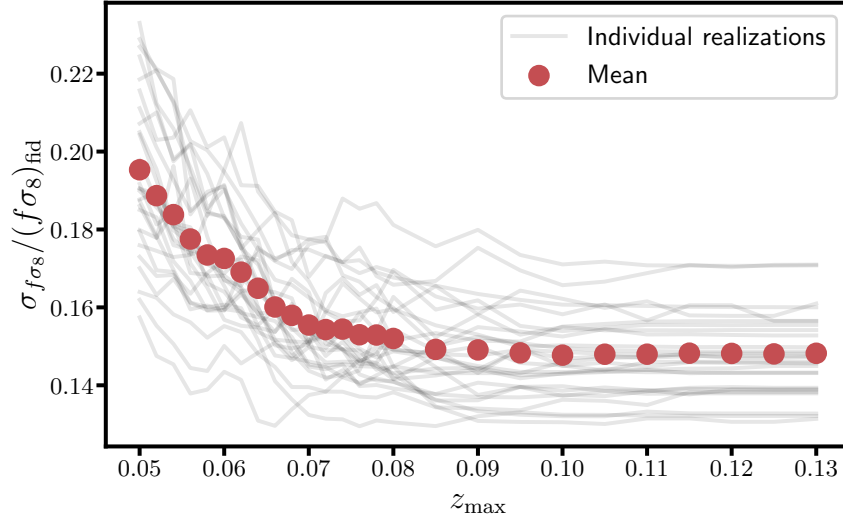


Figure 6.18 – Error obtained on $f\sigma_8$ in the fit with unbiased velocities. The red points represent the average of the errors per redshift interval, the gray lines indicate the errors for each of the 27 realizations.

The velocities obtained in this way are no longer biased, regardless of redshift. The results of the fit of $f\sigma_8$ on these velocities as a function of the cutoff redshift $z_{\max}$ are plotted in Fig. 6.17. We can see that, as expected, the result is no longer biased. In Fig. 6.18, we show the evolution of the error on $f\sigma_8$ as a function of $z_{\max}$. We note that in the fit of $f\sigma_8$ on this unbiased dataset with a redshift cutoff at $z = 0.06$, the precision is 17%, which has therefore decreased compared to the previous fit on the estimated velocities, for which we had obtained a precision of 19%. This decrease can be understood by the fact that in this fit all the biases related to the velocity estimation (see Fig 6.16) are avoided. Between the cutoff redshifts $z_{\max} = 0.06$ and $z_{\max} = 0.13$ the error goes from 17 to 15%. This evolution is fairly small even though the SNe Ia statistics double over the same interval. This small decrease in the error can be explained by at least two factors. The first is that, as seen in Fig. 5.13, the density of SNe Ia decreases sharply after $z \sim 0.06$. The second is that the error on the velocity estimation increases almost linearly with redshift. We can therefore conclude that the additional volume that we include when using SNe Ia at redshifts beyond $z \sim 0.06$ is, due to the spectroscopic selection, poorly sampled and with more noise than at lower redshift. The constraining power of these additional SNe Ia is therefore fairly weak and does not significantly decrease the error on $f\sigma_8$.

6.3.5 Predictions for measuring $f\sigma_8$ with the ZTF DR2 simulation

We reproduced the analysis for our ZTF DR2 simulation. The number of SNe Ia in this simulation is about half that of the six-year ZTF simulation, with $\langle N \rangle = 1658$ without a redshift cutoff and $\langle N \rangle = 775$ with a cutoff at $z_{\max} = 0.06$.

Fig. 6.19 shows the constraints on $f\sigma_8$ for the simulated ZTF DR2 dataset with a redshift cutoff $z_{\text{max}} = 0.06$. The fit with the true velocities gives a result of

$$f\sigma_8/(f\sigma_8)_{\text{fid}} = 0.972 \pm 0.019. \quad (6.39)$$

This result is in tension with the fiducial value at $\sim 1.5\sigma$. This bias is smaller than the average uncertainty

$$\sqrt{\langle \sigma_{f\sigma_8}^2 \rangle}/(f\sigma_8)_{\text{fid}} = 0.128. \quad (6.40)$$

Moreover, if we look at the first panel of Fig. 6.20, which shows the fit results for $f\sigma_8$ as a function of the redshift cutoff, we see that the measurement of $f\sigma_8$ seems to converge toward the fiducial value as more SNe Ia are added. The result for the fit of $\mathbf{p}$ with the estimated velocities and with $\mathbf{p}_{\text{HD}}$ fixed to the input values of the simulation and the same prior on σ_u as before is

$$f\sigma_8/(f\sigma_8)_{\text{fid}} = 0.968 \pm 0.046. \quad (6.41)$$

This result is compatible with the fiducial value of $f\sigma_8$. The average uncertainty obtained is

$$\sqrt{\langle \sigma_{f\sigma_8}^2 \rangle}/(f\sigma_8)_{\text{fid}} = 0.246. \quad (6.42)$$

For the fit with $\mathbf{p}$ and $\mathbf{p}_{\text{HD}}$ free, we obtain

$$f\sigma_8/(f\sigma_8)_{\text{fid}} = 0.923 \pm 0.051. \quad (6.43)$$

This result is in tension at $\sim 1.5\sigma$ with the fiducial value. We note on the bottom panel of Fig. 6.20 that the lower statistics cause instabilities in the fit for a larger number of realizations, which may be partly responsible for low results for $f\sigma_8$. Here too the bias is not significant compared to the average uncertainty

$$\sqrt{\langle \sigma_{f\sigma_8}^2 \rangle}/(f\sigma_8)_{\text{fid}} = 0.255. \quad (6.44)$$

As before, the difference between the errors obtained with or without fixing $\mathbf{p}_{\text{HD}}$ is small.

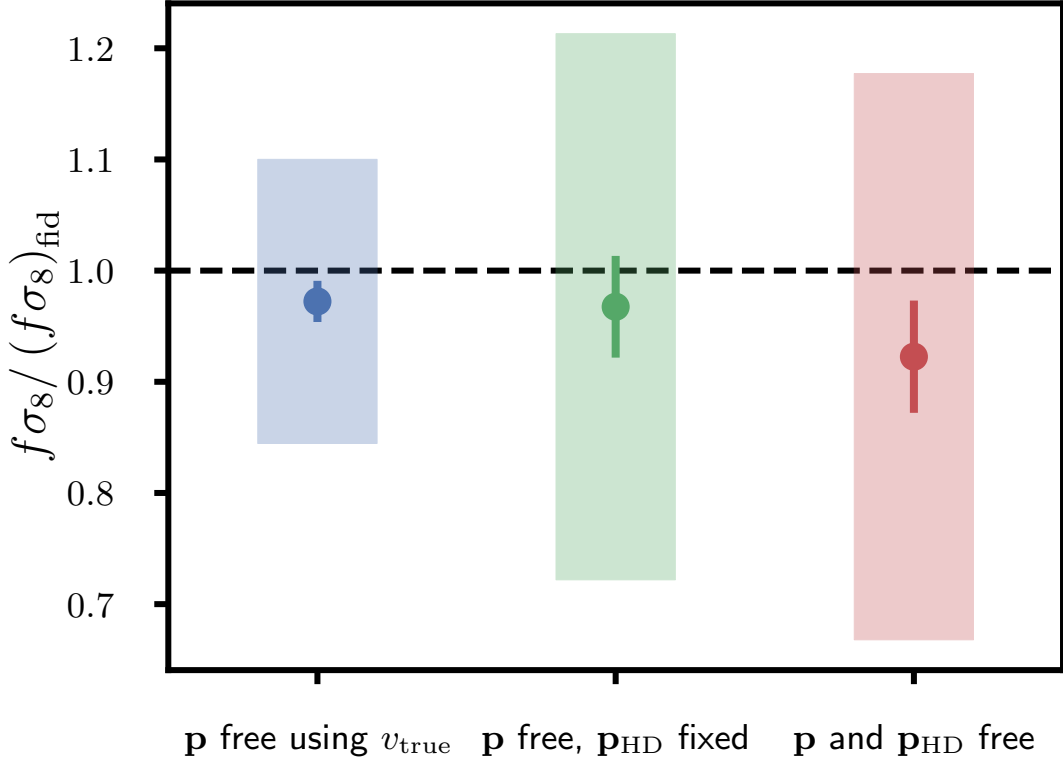


Figure 6.19 – Best constraints on $f\sigma_8$ for the simulated ZTF DR2 dataset with SNe Ia in the redshift interval $z \in [0.02, 0.06]$. The points with error bars show the mean obtained over our 27 realizations, the colored rectangles represent the average uncertainty on $f\sigma_8$.

A comparison with the error obtained previously for the six years of ZTF can give us a first estimate of the share of cosmic variance in the total error on $f\sigma_8$. If we model the error as

$$\sigma_{f\sigma_8} = \sqrt{\frac{\sigma^2}{N} + b^2}, \quad (6.45)$$

where σ^2 is a component whose importance decreases as the number of SNe Ia increases, and b a constant component related to cosmic variance. With two measurements of the growth rate of structures in the same redshift interval and with different numbers of SNe Ia, we can obtain the cosmic variance component as

$$b = \sqrt{\frac{N_1 \sigma_{f\sigma_8,1}^2 - N_2 \sigma_{f\sigma_8,2}^2}{N_1 - N_2}}, \quad (6.46)$$

with N_1, N_2 the numbers of SNe Ia and $\sigma_{f\sigma_8,1}, \sigma_{f\sigma_8,2}$ the errors on $f\sigma_8$ for, respectively, each of the two constraints. If we apply Eq. 6.46 to the results obtained with the true

velocities we get

$$\sqrt{\langle b^2 \rangle} / (f\sigma_8)_{\text{fid}} = \sqrt{\frac{1620 \times 0.100^2 - 775 \times 0.128^2}{1620 - 775}} \simeq 0.064. \quad (6.47)$$

This value of b seems to indicate that the ZTF dataset will not be limited by cosmic variance even after six years of data taking. We can estimate that the statistical component will have the same amplitude as the cosmic variance for $N \sim 2300$ SNe Ia. This estimate is only valid for a dataset limited to $z < 0.06$ and remains simplistic. It could be refined in future work.

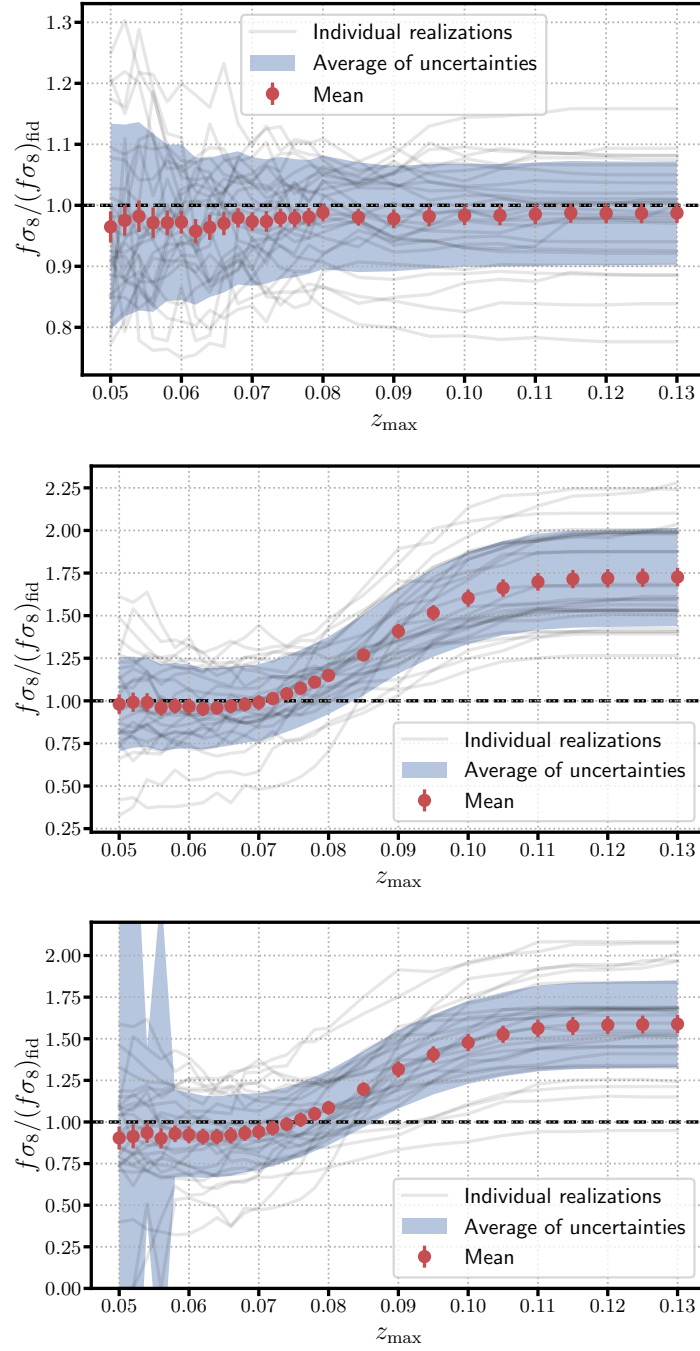


Figure 6.20 – Results of the fit of $f\sigma_8$ as a function of the cutoff redshift z_{max} for the 27 realizations of the DR2 dataset. The red points represent the mean value of $f\sigma_8$ for each z_{max} , the blue band indicates the average uncertainty obtained on $f\sigma_8$, and the gray lines indicate the results for each of the 27 realizations. *Top*: Results of the fit on the true velocities v_{true} . *Center*: Results of the fit on the estimated velocities $\hat{v}$ with $\mathbf{p}$ free and $\mathbf{p}_{\text{HD}}$ fixed. *Bottom*: Results of the fit on the estimated velocities $\hat{v}$ with $\mathbf{p}$ and $\mathbf{p}_{\text{HD}}$ free.

6.3.6 Comparison with existing and future measurements

We can compare our result obtained for the equivalent of six years of ZTF with current measurements. In Sec. 6.3.1, we obtained a constraint with 19% error on $f\sigma_8$ using $\langle N \rangle \sim 1660$ SNe Ia in a redshift interval $0.02 < z < 0.06$. Fig. 6.21 compares the various existing measurements of $f\sigma_8$ with the one predicted for ZTF in this thesis work. Some of these constraints use, in addition to peculiar velocities, density tracers to compute the velocity-density cross-correlation and improve the constraint. The works that use only peculiar velocities are those of Johnson et al. 2014; Howlett et al. 2017a. In Johnson et al. 2014, the use of 8896 distances obtained via the FP relation from the 6dF catalog in a redshift interval $0 < z < 0.05$, combined with 203 SNe Ia distances at $z < 0.07$ from various datasets, allowed a constraint on $f\sigma_8$ with a precision of $\sim 15\%$. Howlett et al. 2017a uses 2062 distances obtained via the TF relation from the 2MTF catalog in a redshift interval $0.002 < z < 0.03$ to constrain $f\sigma_8$ with a precision of $\sim 16\%$. Our prediction is comparable to these two measurements, although somewhat less precise. However, ZTF samples a larger volume $0.02 < z < 0.06$ over the entire northern hemisphere, which should make it less sensitive to cosmic variance on $f\sigma_8$.

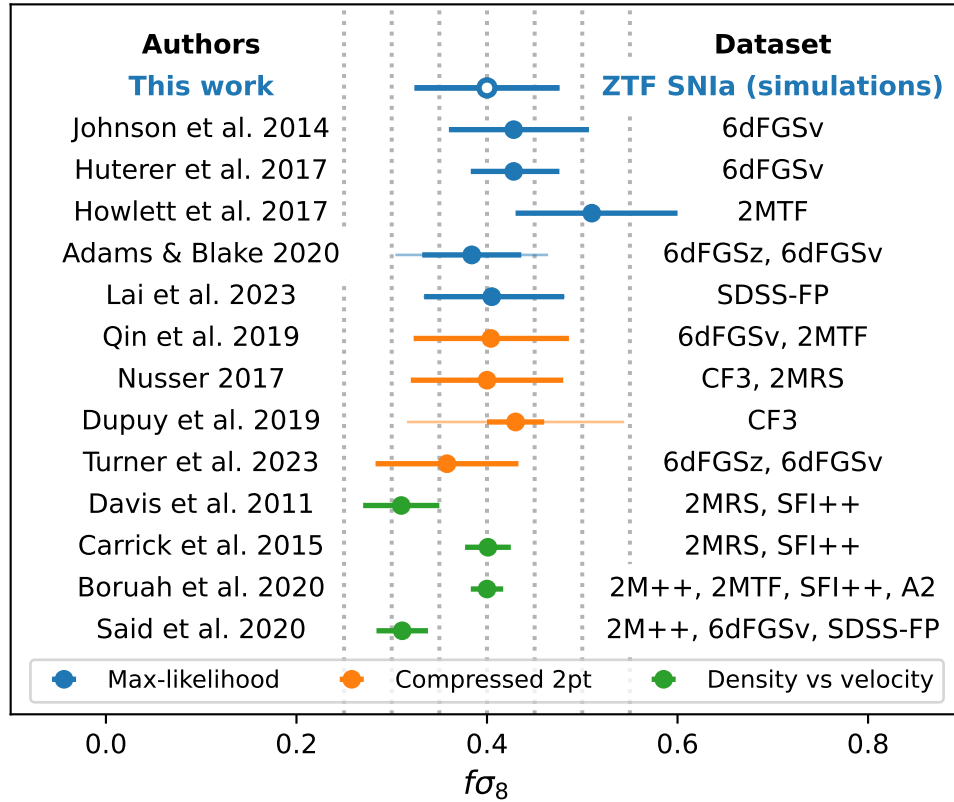


Figure 6.21 – Comparison between the constraint on $f\sigma_8$ predicted for ZTF in our work and existing constraints.

Another interesting comparison is with the predictions for future direct FP and TF measurements from DESI (Saulder et al. 2023). We have plotted the forecasts for these measurements in Fig. 6.22. In the redshift interval $0 < z < 0.05$, which is similar to that of ZTF, the best forecast for DESI’s peculiar velocities is a precision of $\sim 16\%$ on $f\sigma_8$. DESI is expected to measure a total of ~ 133000 FP and ~ 53000 TF distances usable for cosmology across all redshifts.

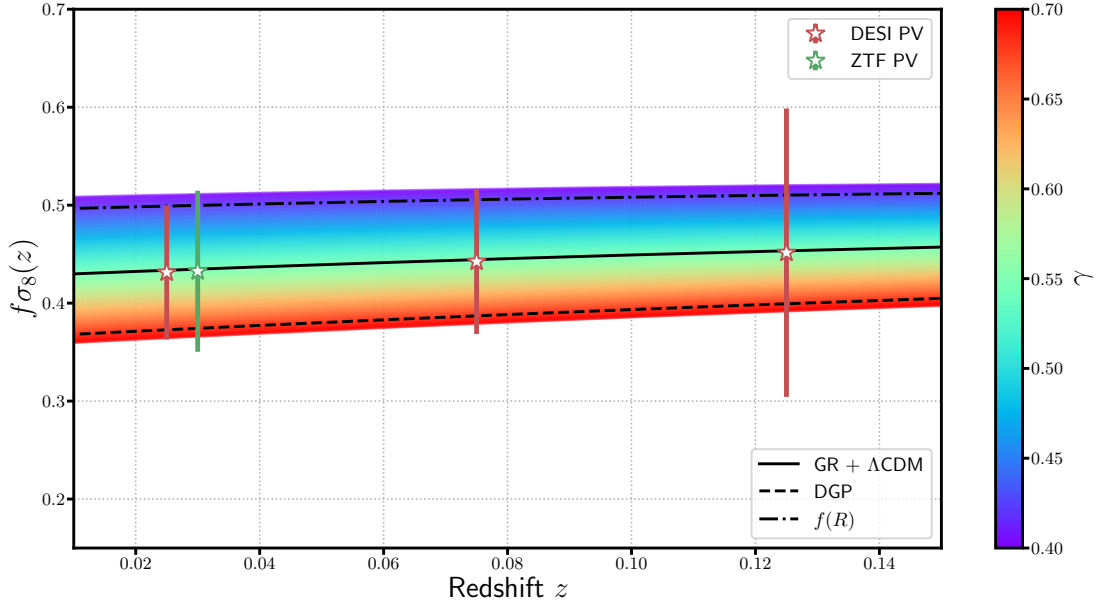


Figure 6.22 – Forecasts for the constraint on $f\sigma_8$ for the peculiar velocities of ZTF (Carreres et al. 2023) and of DESI (Saulder et al. 2023).

Although the forecasts for the ZTF constraints still seem somewhat below what is expected for the DESI galaxy TF and FP surveys, it should be noted that our work focused on SNe Ia classified spectroscopically. Work is underway to build a dataset with a photometric classification, which should allow an increase in the statistics.

Conclusion

General conclusion

One of the challenges of modern cosmology is to understand the origin of the accelerated expansion of the Universe. We must therefore discriminate between the possibility of a new form of energy, called dark energy, represented by the cosmological constant Λ in the standard model, and that of a theory of gravity beyond general relativity. Observing large-scale matter structures, and in particular their evolution over time, makes it possible to draw this distinction.

My work focused on measuring the growth rate of structure $f\sigma_8$ within the framework of the ZTF survey. I developed a complete simulation of a SNe Ia survey that uses an N -body simulation to obtain a cosmological velocity field, together with observational data to obtain realistic light curves. I then applied to the simulated data the photometric and spectroscopic selections specific to the ZTF survey. This allowed me to study the selection bias linked to the classification limit for the faintest SNe Ia. This bias directly impacts the residuals of the Hubble diagram and becomes significant beyond *redshift* $z > 0.06$. The estimation of velocities from the Hubble diagram residuals is likewise biased beyond the same *redshift*. I used the maximum likelihood method on a simulated dataset corresponding to 6 years of the ZTF survey in order to constrain the parameter $f\sigma_8$. The results show that, by restricting the sample to SNe Ia with *redshift* $z_{\max} < 0.06$, one can obtain a constraint on $f\sigma_8$ with an error of 19% without significant bias. This result is already comparable to those obtained with a similar method using several thousand galaxies, and to the predictions for the DESI survey. Another result is that including SNe Ia at higher *redshift* leads to a significant bias on $f\sigma_8$ dominated by the spectroscopic selection. This bias arises because the likelihood function used does not correctly model the distribution of biased velocities. We quantified that a perfect correction of this bias, although doubling the SNe Ia statistics, does not lead to a significant improvement in the constraint on $f\sigma_8$. This effect is understood as a consequence of the rapid decrease in the density of observed SNe Ia beyond $z \sim 0.06$ and of the near-linear increase in the velocity estimation error with *redshift*. As a result, data at *redshift* $z > 0.06$ have little constraining power.

Outlook

The next step in this work is to apply the maximum likelihood method to the ZTF SNe Ia data classified spectroscopically. However, the *redshifts* of the SNe Ia host

galaxies are not yet all available with spectrometric precision. The DESI survey is expected to provide most of them.

The possibility of increasing the SNe Ia statistics through the use of photometric classification methods is currently being investigated. The systematics related to data contamination by non-Ia supernovae must be assessed within the framework of the $f\sigma_8$ analysis. Photometric classification of SNe Ia could make it possible to increase the statistics and push back the *redshift* limit associated with the selection bias.

The simulation method proposed in this thesis can also be improved. The noise model could be refined to more realistically account for measurement noise, and to verify that the noise sources neglected in this work do not result in significant biases or errors on $f\sigma_8$. In addition, the effects of the intrinsic dispersion of SNe Ia should be evaluated in the case of a wavelength-dependent dispersion. Indeed, recent work seems to show a dependence between SNe Ia color and the Hubble diagram residuals (Brout and Scolnic 2021), which could affect the estimation of peculiar velocities. More generally, studies remain to be carried out on the correlations between the properties of SNe Ia and those of their host galaxies. These correlations could affect the velocity distribution and have an impact on the constraint on $f\sigma_8$. One can, for example, consider correlations with host galaxy mass, since the velocity distribution of galaxies may differ depending on whether they are more or less massive. These effects will need to be included in future simulations.

Work is underway to extend the $f\sigma_8$ analysis to future SNe Ia data that will be observed by the Vera Rubin Observatory. Predictions based on Fisher matrices are already available (Howlett et al. 2017b), but the analysis still needs to be studied in more detail using simulations.

Future analyses combining velocities derived from the TF or FP relations of galaxies, for instance those of the DESI survey, with those obtained from ZTF SNe Ia are also in preparation. Combining these datasets could increase the statistics and thus the constraint on $f\sigma_8$. Finally, velocity-density cross-correlation analyses are also being explored and could lead to the best constraints on $f\sigma_8$ at low *redshift*, in particular through the joint use of ZTF and DESI data, or Vera Rubin and 4MOST data.

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